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Name Answer Key

PreCalculus; Quiz 1.1 – 1.3

1. Write the equation of a line, in point-slope form, through the given point with the given slope. (1 Point)

$$(1, 2) \quad m = 3$$

$\boxed{y - 2 = 3(x-1)}$

2. Find the domain of the function. Write your answer in ***set-builder notation***. (2 Points Each)

a. $h(t) = \frac{4}{t}$

$t \neq 0$

$D: \{ t \,/\, t \neq 0 \}$

b. $g(x) = \frac{1}{x} - \frac{3}{x+2}$

$x \neq 0, -2$

$D: \{ x \,|\, x \neq 0, -2 \}$

3. Determine if the function is *even, odd*, or *neither:* $k(x) = x^2 - 4$. (2 Points)

$k(-x) = (-x)^2 - 4$

$= x^2 - 4$

$\boxed{\text{EVEN}}$

4. Evaluate the difference quotient $\left(\frac{f(x+h)-f(x)}{h}\right)$ for the following function: $f(x) = 4x - 1$. (3 Points)

$\frac{f(x+h) - f(x)}{h}$

$= \frac{[4(x+h) - 1] - [4x - 1]}{h}$

$= \frac{4x + 4h - 1 - 4x + 1}{h}$

$= \frac{4h}{h}$

$\boxed{= 4}$

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5. Find the interval(s) where the function is *increasing, decreasing,* or *constant.* Also, state the *domain and range* of the function. All answers should be written in ***interval notation***. (5 Points)

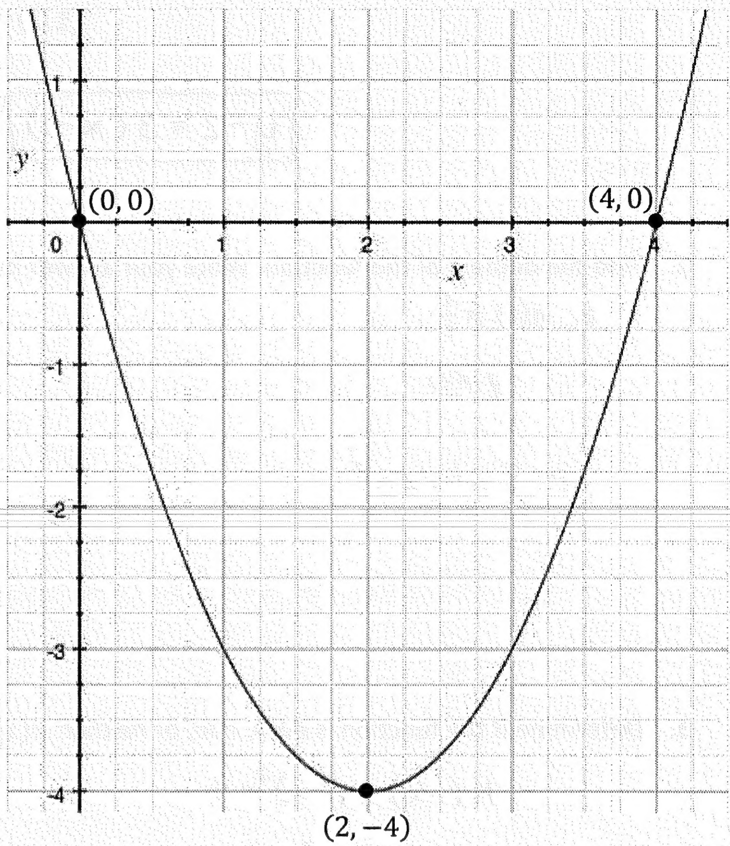
increasing: $(2, +\infty)$

decreasing: $(-\infty, 2)$

constant: N/A

domain: $(-\infty, +\infty)$

range: $[-4, +\infty)$



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Name Answer Key

Pre-Calculus; Vocabulary Test – Chapter 1

Matching: Match each word with its definition. (1 Point Each)

G 1. Parallel lines	A. $f(-x) = -f(x)$
J 2. Perpendicular lines	B. lowest point in a section of a graph
A 3. Odd function	C. x values
I 4. Even function	D. the rate of change of a line
F 5. Relative maximum	E. y values
B 6. Relative minimum	F. highest point in a section of a graph
C 7. Domain	G. lines with the same slope
E 8. Range	H. uses ( ) and/or [ ] to indicate domain and range
D 9. Slope	I. $f(-x) = f(x)$
K 10. Function	J. lines with opposite reciprocal slopes
H 11. Interval Notation	K. special relation where one input gives exactly one output

Multiple Choice: Choose the answer that best completes the statement and write the letter on the blank to the left of the problem number. (1 Point Each)

B 12. A line that is parallel to $y = -6x - 5$ through the point (2,1) is ______________.

a. $y = -6x + 5$ b. $y = -6x + 13$ c. $y = \frac{1}{6}x + 5$

d. $y = \frac{1}{6}x + 13$ e. $y = 6x + 5$

C 13. Two other vocabulary words that have the same meaning as *independent variable* are ________.

a. Domain and Range b. Range and Output c. Domain and Input

d. Input and Output e. Domain and Output

C 14. If $f(x) = 2x - 3$ and $g(x) = 2$ then $f(g(x)) =$ ________.

a. -1 b. 0 c. 1 d. 2 e. 3

A 15. What is the domain of the function $f(x) = |x + 3|$?

a. $(-\infty, \infty)$ b. $(3, \infty)$ c. $[3, \infty)$ d. $(-3, \infty)$ e. $[-3, \infty)$

C 16. If something is *not* a function, it is a(n) _______________.

a. Unfunction b. Reciprocal c. Relation d. Reflection e. Odd

A 17. Another way to write $(f \circ g)(x)$ is __________.

a. $f(g(x))$ b. $g(f(x))$ c. $(fg)(x)$ d. $f(x)g(x)$ e. fgx

Answer each question or solve each problem below. Remember to show all of your work!

1. Determine if the function $f(x) = x^2 + 2x - 1$ is even, odd or neither. *Prove* your answer. (3 Points)

$f(-x) = (-x)^2 + 2(-x) - 1 = x^2 - 2x - 1$

$\boxed{\text{NEITHER}}$

2. Let $f(x) = 2x^2 - 5$ and $g(x) = x - 3$, find a-e. Simplify your answers completely. Give the domain when appropriate. (6 Points)

   a. $(f+g)(x) = 2x^2 - 5 + x - 3$

   $\boxed{= 2x^2 + x - 8}$

   b. $(f-g)(x) = 2x^2 - 5 - x + 3$

   $\boxed{= 2x^2 - x - 2}$

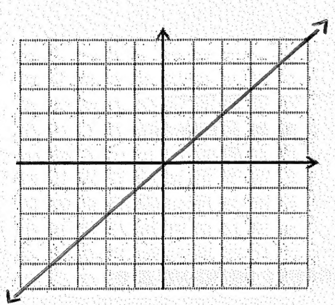
   c. $(fg)(x) = (2x^2 - 5)(x - 3)$

   $\boxed{= 2x^3 - 6x^2 - 5x + 15}$

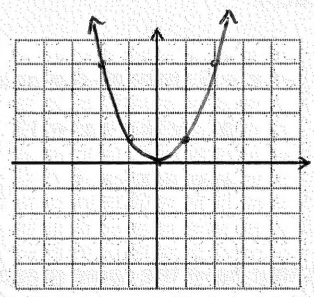
   d. $\left(\frac{f}{g}\right)(x) = \boxed{\frac{2x^2 - 5}{x - 3},\ x \neq 3}$

   e. $(f+g)(5) = 47$

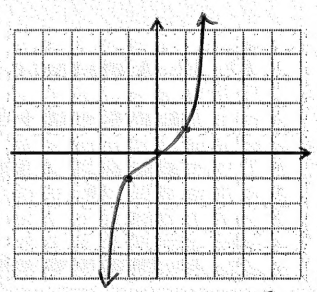
3. Draw the six parent functions and label them with their name *or* equation. (6 Points)



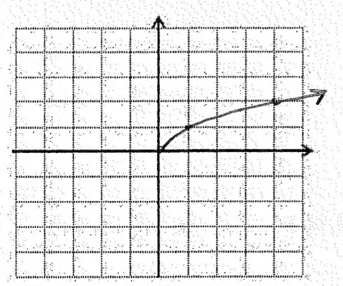
LINEAR $p(x) = x$



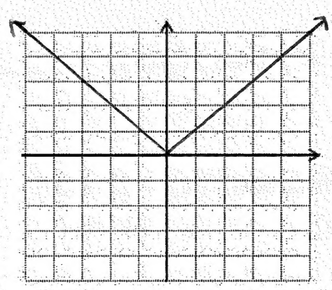
QUADRATIC $p(x) = x^2$



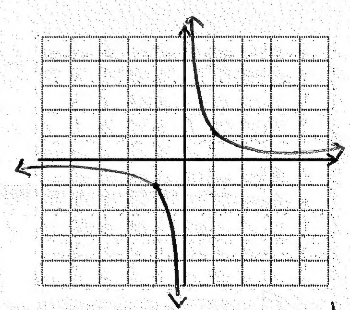
CUBIC $p(x) = x^3$



SQUARE ROOT $P(x) = \sqrt{x}$



ABSOLUTE VALUE $p(x) = |x|$



RATIONAL $p(x) = 1/x$

4. Describe (a) what the parent function is, (b) the transformations/shifts that happen and (c) sketch a graph of the new function on the axis. (3 Points Each)

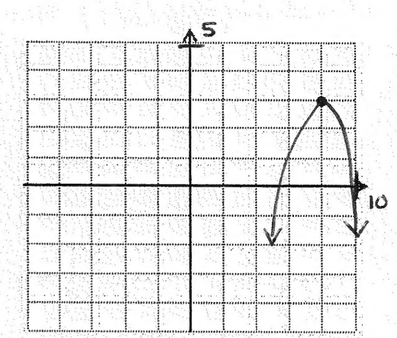
   i. $g(x) = 3 - (x - 8)^2$

   a) Quadratic

   $p(x) = x^2$

   B) Shift → 8, reflect y-axis,

   Shift ↑ 3

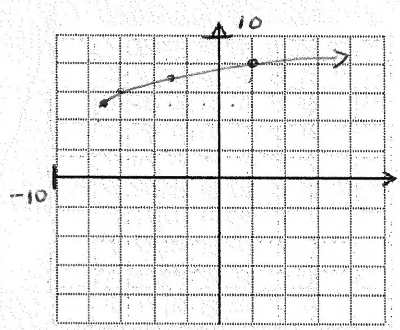


   ii. $h(x) = \sqrt{x + 7} + 5$

   a) SQUARE ROOT

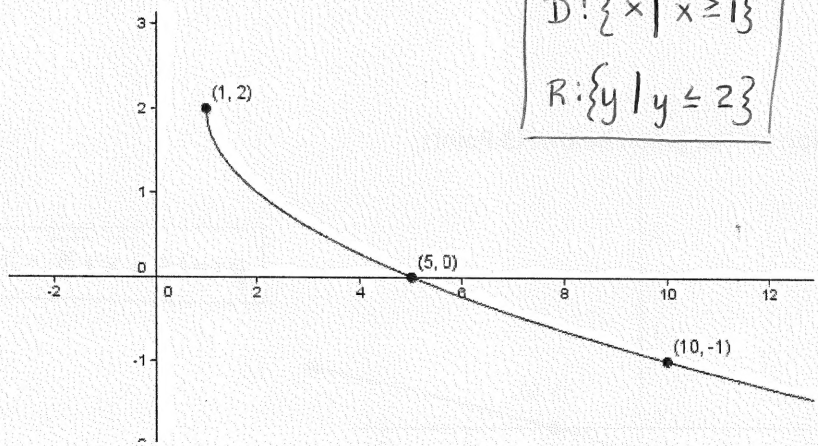
   $p(x) = \sqrt{x}$

   B) SHIFT ← 7, ↑ 5



5. Find the domain and range of the graph below. Give your answer in **set-builder notation**. (2 Points)

$$\boxed{\begin{array}{l} D: \{x \mid x \geq 1\} \\ R: \{y \mid y \leq 2\} \end{array}}$$



6. Find the inverse of the function $f(x) = \sqrt{4x - 3}$. (2 Points)

$x = \sqrt{4y - 3}$

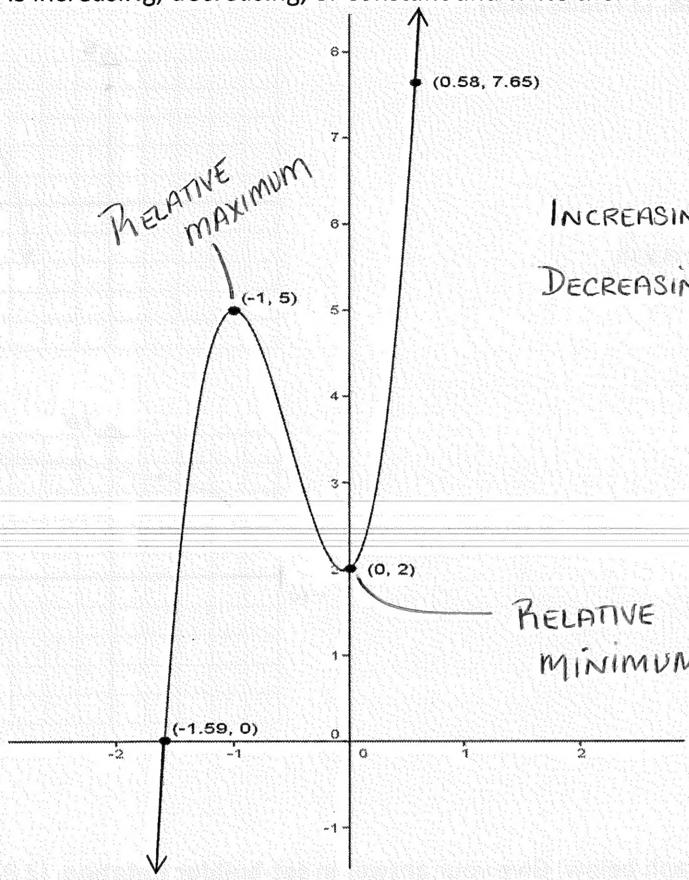
$x^2 = 4y - 3$

$4y = x^2 + 3$

$y = \dfrac{x^2 + 3}{4}$

$$\boxed{f^{-1}(x) = \frac{x^2 + 3}{4}}$$

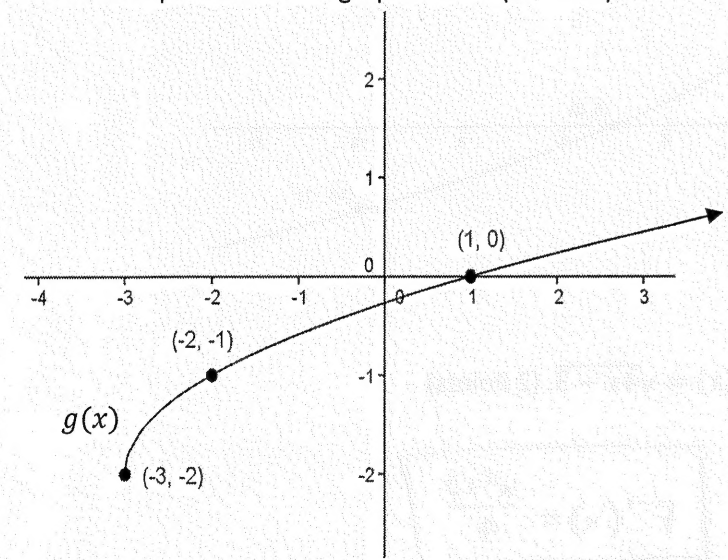
7. Label the relative maximum and/or relative minimum. Then determine the intervals where the function is increasing, decreasing, or constant and write them in **interval notation**. (4 Points)



INCREASING: $(-\infty, -1)$ $(0, +\infty)$

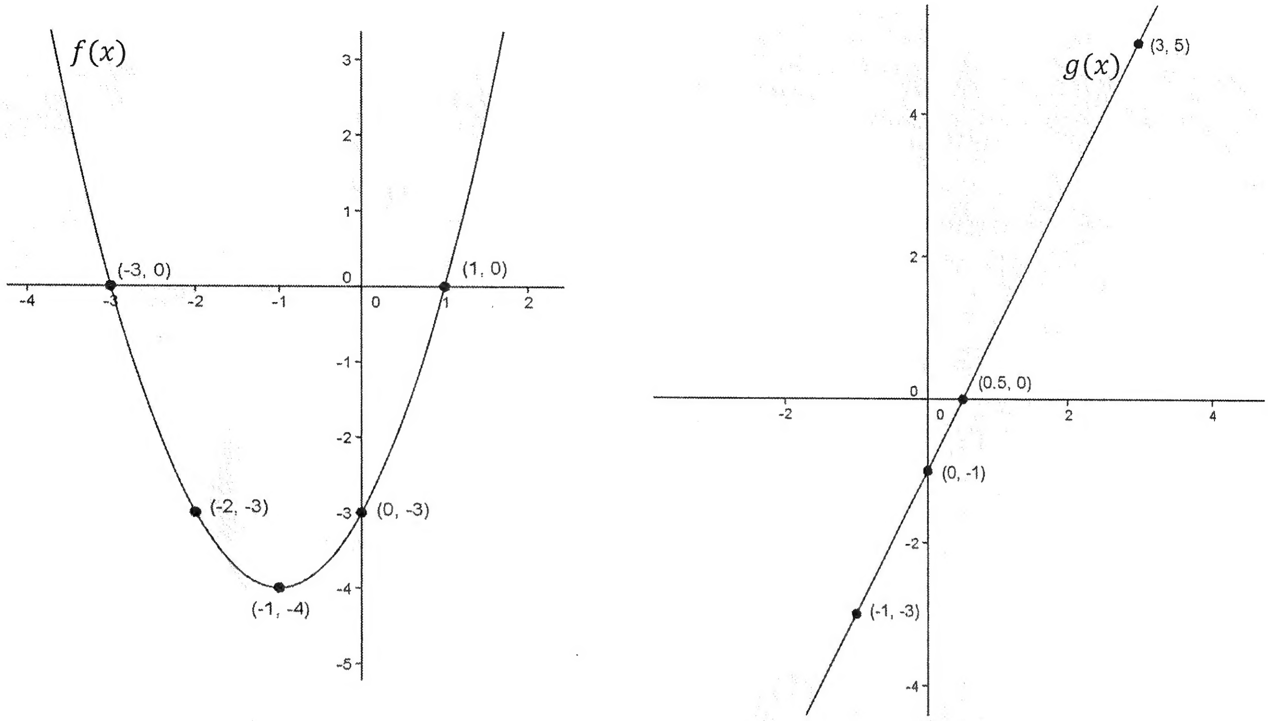
DECREASING: $(-1, 0)$

8. Write an equation for the graph below. (3 Points)



$\boxed{p(x) = \sqrt{x+3} - 2}$

9. Using the two functions below, evaluate the compositions given in i - iii. (1 Point Each)



i. $g(f(1)) =$ -1

ii. $f(g(0)) =$ -4

iii. $f(f(-3)) =$ -3

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Name Answer Key

PreCalculus; Quiz 2.1 – 2.2

Determine the vertex of the following functions. (1 Point Each)

1. $f(x) = (x + 4)^2 - 3$

VERTEX

(-4, -3)

2. $h(x) = x^2 - 8x + 16$

$= (x-4)^2$

VERTEX

(4, 0)

Find the x-intercepts of the function. Then find the relative extrema of the function and identify if it is a maximum or minimum. (4 Points)

3. $g(x) = 2x^2 - 5x + 3$

$= (2x-3)(x-1)$

X-INTERCEPTS

$(\frac{3}{2}, 0)(1, 0)$

VERTEX

$x = \frac{5}{4}$

$y = \frac{-1}{8}$

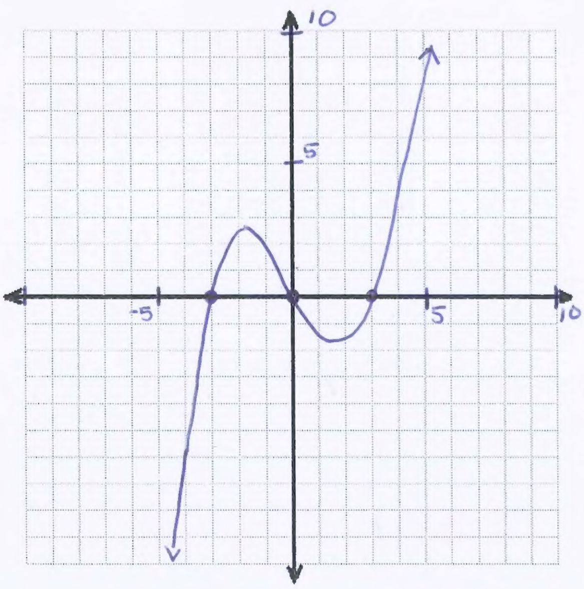
$(\frac{5}{4}, \frac{-1}{8})$

(1.25, -0.125)

MINIMUM

Sketch the graph of the polynomial function. ** Include what the arms are doing, and any x- or y-intercpets** (2 Points Each

4. $f(x) = x^3 - 9x$



L.C. → 1

exp → 3

↑

↓

X-INT.

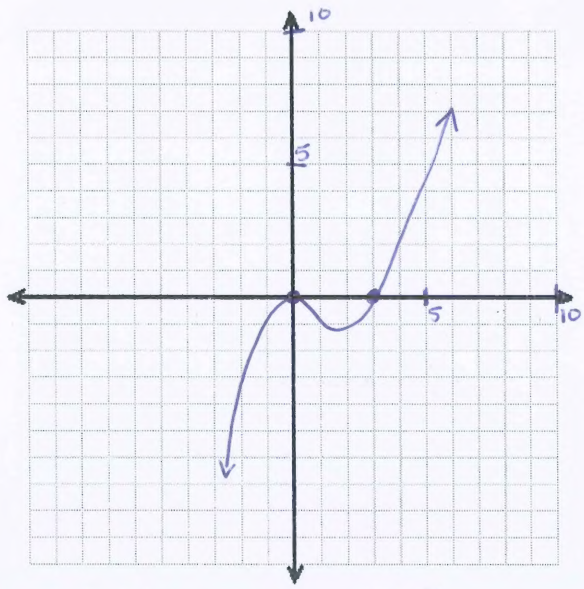
$x(x^2-9)=0$

$x = 0, \pm 3$

Y-INT

(0, 0)

5. $g(x) = x^3 - 3x^2$



L.C. → 1

exp → 3

↑

↓

X-INT

$x^2(x-3)=0$

$x = 0, 3$

Y-INT

(0, 0)

/10

Name ANSWER KEY

PreCalculus; Quiz 2.3 – 2.4

Use either long division or synthetic division to simplify the two polynomials. (2 Points Each)

1. $(4x^3 - 7x^2 - 11x + 5) \div (4x + 5)$

$x^2 - 3x + 1$

$4x+5 \,\overline{)\, 4x^3 - 7x^2 - 11x + 5}$

$-(4x^3 + 5x^2)$

$-12x^2 - 11x + 5$

$-(-12x^2 - 15x)$

$4x + 5$

$-(4x + 5)$

0

$\boxed{x^2 - 3x + 1}$

OR

$-\frac{5}{4} \mid 4 \quad -7 \quad -11 \quad 5$

$-5 \quad 15 \quad -5$

$4 \quad -12 \quad 4 \quad 0$

$\boxed{4x^2 - 12x + 4}$

2. $(x^4 + 5x^3 + 6x^2 - x - 2) \div (x + 2)$

$-2 \mid 1 \quad 5 \quad 6 \quad -1 \quad -2$

$-2 \quad -6 \quad 0 \quad 2$

$1 \quad 3 \quad 0 \quad -1 \quad 0$

$\boxed{x^3 + 3x^2 - 1}$

$x^3 + 3x^2 - 1$

$x+2 \,\overline{)\, x^4 + 5x^3 + 6x^2 - x - 2}$

$-(x^4 + 2x^3)$

$3x^3 + 6x^2 - x - 2$

$-(3x^3 + 6x^2)$

$-x - 2$

$-(-x - 2)$

0

3. $(7x^3 + 3) \div (x + 2)$

$-2 \mid 7 \quad 0 \quad 0 \quad 3$

$7 \quad 28 \quad -56$

$7 \quad -14 \quad 28 \quad -53$

$\boxed{7x^2 - 14x + 28 - \frac{53}{x+2}}$

$7x^2 - 14x + 28$

$x+2 \,\overline{)\, 7x^3 + 0x^2 + 0x + 3}$

$-(7x^3 + 14x^2)$

$-14x^2 + 0x + 3$

$-(-14x^2 - 28x)$

$28x + 3$

$-(28x + 56)$

-53

Simplify the expression. Write the answer in standard form. (2 Points Each)

4. $(4 + i) - (7 - 2i)$

$-3 + 3i$

5. $(6 - 2i)(2 - 3i)$

$12 - 18i - 4i + 6i^2$

$\boxed{6 - 22i}$

Name Answer Key

Pre-Calculus; Vocabulary Test – Chapter 2

Spelling. Spell each of the words correctly. (1 Point Each)

1. Asymptote
2. Horizontal
3. Vertical
4. Quadratic
5. Continuous

Multiple Choice. Choose the answer that best completes the statement and write the letter on the blank to the left of the problem number. (1 Point Each)

A 6. The graph of a quadratic function is a special "U-shaped" curve called a(n) ____________.

a. Parabola b. Hyperbola c. Ellipse d. Conic Section

B 7. According to your textbook, the standard form of a quadratic function is ____________.

a. $y = ax^2 + bx + c$ b. $y = a(x-h)^2 + k$ c. $y = x^2$ d. $y = \sqrt{x}$

D 8. One feature of a polynomial function is the graph contains smooth, rounded turns. The other feature of a polynomial function is that it is ____________.

a. a line b. not continuous c. has holes d. continuous

C 9. Which of the following expressions *cannot* be used in synthetic division?

a. $x - 2$ b. $(x-2)(x+3)$ c. $x^2 + 1$ d. x

C 10. The imaginary unit, i, is equal to which of the following expressions: ____________.

a. -1 b. 1 c. $\sqrt{-1}$ d. $-i$

A 11. Which of the following expressions is the conjugate of $2 + 3i$?

a. $2 - 3i$ b. $-2 + 3i$ c. $-2 - 3i$ d. $2 + 3i$

D 12. The ____________ of a fraction, when set equal to zero, will give the vertical asymptotes of a rational function.

a. Numerator b. Quotient c. Remainder d. Denominator

Graph the given quadratic function by first finding the vertex (by completing the square or using $x = -\frac{b}{2a}$), then plotting 2 extra points. (3 Points)

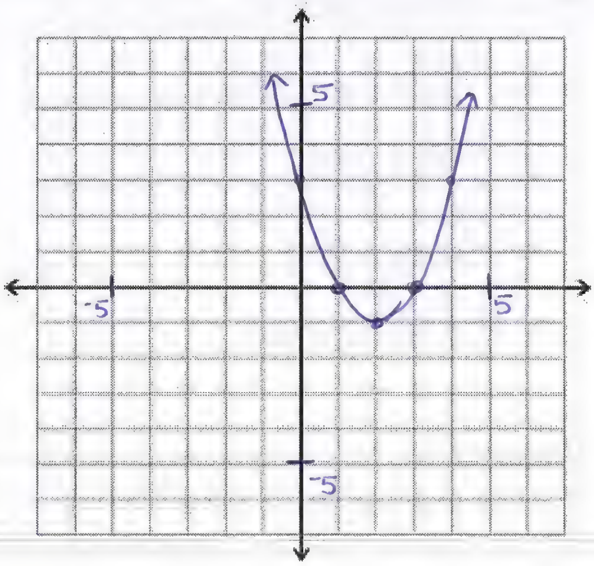
1. $f(x) = x^2 - 4x + 3$

$x = \frac{4}{2} = 2$

$f(2) = 4 - 8 + 3 = -1$

$\boxed{\text{VERTEX } (2,-1)}$

x	y
1	0
3	0



Identify if the following functions have a minimum or maximum. Then find the minimum or maximum value. (2 Points Each)

2. $f(x) = -x^2 + 2x + 5$

$x = \frac{-2}{-2} = 1$

$f(1) = -1 + 2 + 5 = 6$

$\boxed{\text{MAXIMUM } (1,6)}$

3. $f(x) = x^2 + 4x + 1$

$x = \frac{-4}{2} = -2$

$f(-2) = 4 - 8 + 1 = -3$

$\boxed{\text{MINIMUM } (-2,-3)}$

Graph the given polynomial using (a) the Leading Coefficient Test, (b) finding the x-intercepts, (c) plotting a few extra points, and (d) drawing a smooth curve. (4 Points)

4. $f(x) = -2x^3 - x^2 + x$

a) L.C. → −2 ⟹ ↑ ↓
EXP → ODD

B) X-INTERCEPT ($y = 0$)

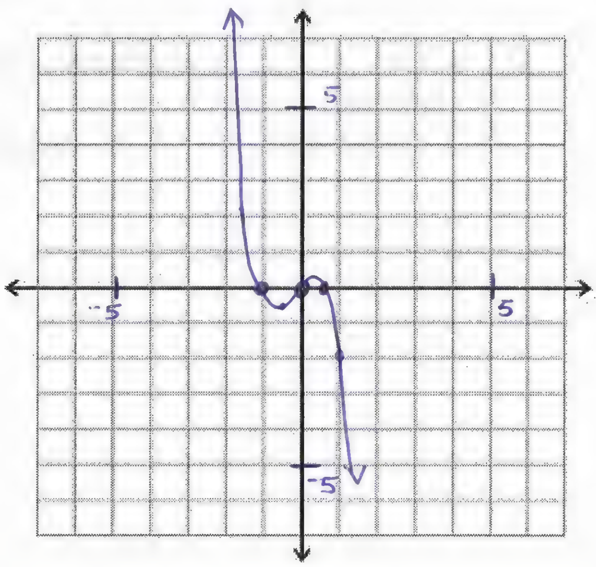
$-x(2x^2 + x - 1) = 0$

$-x(2x - 1)(x + 1) = 0$

$\boxed{(0,0)\ (\tfrac{1}{2},0)\ (-1,0)}$

c)

x	y
−2	10
−0.5	−0.5
1	−2



Find a fourth degree polynomial that satisfies the following conditions. (3 Points)

5. Zero: -2, multiplicity: 2; Zero: 3, multiplicity: 2; Falls to the Left, Falls to the Right

$p(x) = -(x+2)^2(x-3)^2$

$= -(x^2+4x+4)(x^2-6x+9)$

$= -(x^4-6x^3+9x^2+4x^3-24x^2+36x+4x^2-24x+36)$

$\boxed{= -x^4+2x^3+11x^2-12x-36}$

Use long division or synthetic division to divide the polynomials. (2 Points Each)

6. $(24x^2 - x - 8) \div (3x - 2)$

$$\begin{array}{r} 8x+5 \\ 3x-2\overline{)24x^2 - x - 8} \\ -(24x^2-16x) \\ \hline 15x-8 \\ -(15x-10) \\ \hline 2 \end{array}$$

$$\begin{array}{c|ccc} \frac{2}{3} & 24 & -1 & -8 \\ & & 16 & 10 \\ \hline & 24 & 15 & 2 \end{array}$$

$\boxed{8x+5+\frac{2}{3x-2}}$ OR $\boxed{24x+15+\frac{2}{x-\frac{2}{3}}}$

7. $(3x^4 + x^2 - 1) \div (x^2 - 1)$

$$\begin{array}{r} 3x^2+4 \\ x^2+0x-1\overline{)3x^4+0x^3+x^2+0x-1} \\ -(3x^4+0x^3-3x^2) \\ \hline 4x^2+0x-1 \\ -(4x^2+0x-4) \\ \hline 3 \end{array}$$

$\boxed{3x^2+4+\frac{3}{x^2-1}}$

$$\begin{array}{c|ccccc} 1 & 3 & 0 & 1 & 0 & -1 \\ & & 3 & 3 & 4 & 4 \\ \hline -1 & 3 & 3 & 4 & 4 & 3 \\ & & -3 & 0 & -4 & \\ \hline & 3 & 0 & 4 & 0 & \end{array}$$

$\boxed{3x^2+4+\frac{3}{x^2-1}}$

Perform the indicated operation. (1 Point Each)

8. $(8 - 3i) - (-2 + 3i)$

$10-6i$

9. $(\sqrt{5} - \sqrt{12}i)(\sqrt{5} + \sqrt{12}i)$

$5 + \cancel{i\sqrt{60}} - \cancel{i\sqrt{60}} - 12i^2$

$\boxed{17}$

Solve the quadratic equation. (2 Points Each)

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

10. $x^2 + 81 = 0$

$x^2 = -81$

$\boxed{x = \pm 9i}$

11. $x^2 + 4x + 8 = 0$

$x = \frac{-4 \pm \sqrt{16-32}}{2}$

$= \frac{-4 \pm \sqrt{-16}}{2} = \frac{-4 \pm 4i}{2}$

$\boxed{= -2 \pm 2i}$

Find all the zeros of the polynomial function and write it as a product of its linear factors. (3 Points Each)

12. $f(x) = x^5 - 5x^3 + 4x$

$x(x^4 - 5x^2 + 4) = 0$

$x(x^2-1)(x^2-4) = 0$

$\boxed{x = 0, \pm 1, \pm 2}$

$\boxed{f(x) = x(x-1)(x+1)(x+2)(x-2)}$

13. $f(x) = x^4 + 2x^2 - 8$

$(x^2+4)(x^2-2) = 0$

$\boxed{x = \pm\sqrt{2}, \pm 2i}$

$\boxed{f(x) = (x+\sqrt{2})(x-\sqrt{2})(x+2i)(x-2i)}$

14. Find a third degree polynomial with the following zeros: $-3, 4i$. (3 Points)

$p(x) = (x+3)(x-4i)(x+4i)$

$= (x+3)(x^2+16)$

$\boxed{= x^3 + 3x^2 + 16x + 48}$

Graph the rational function by (a) finding all intercepts, (b) finding all asymptotes, (c) plotting a few extra points, and (d) drawing the curve. (4 Points)

15. $f(x) = \frac{3x^2}{x^2-9}$

a) Y-INTERCEPT $(x=0)$

$(0,0)$

X-INTERCEPT $(y=0)$

$(0,0)$

B) VERTICAL Asym.

$x^2 - 9 = 0$

$\boxed{x = \pm 3}$

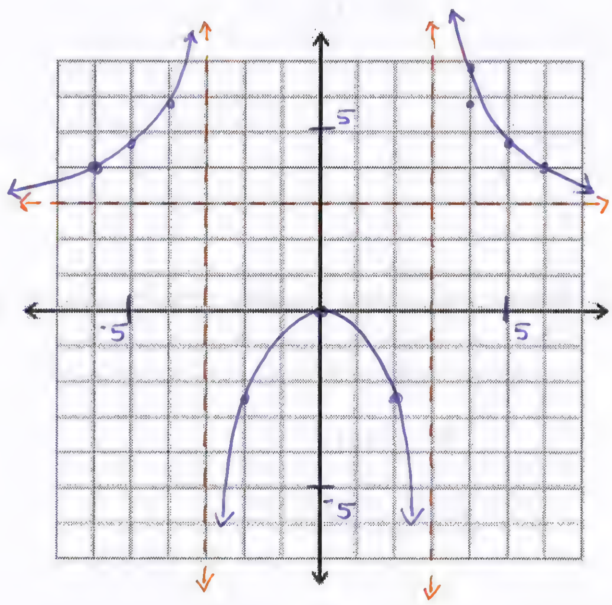
HORIZONTAL Asym.

$n = m$

$y = \frac{3x^2}{x^2} \rightarrow \boxed{y = 3}$

c)

x	y
-6	4
-5	4.7
-4	6.9
-2	-2.4
0	0
2	-2.4
4	6.9
5	4.7
6	4

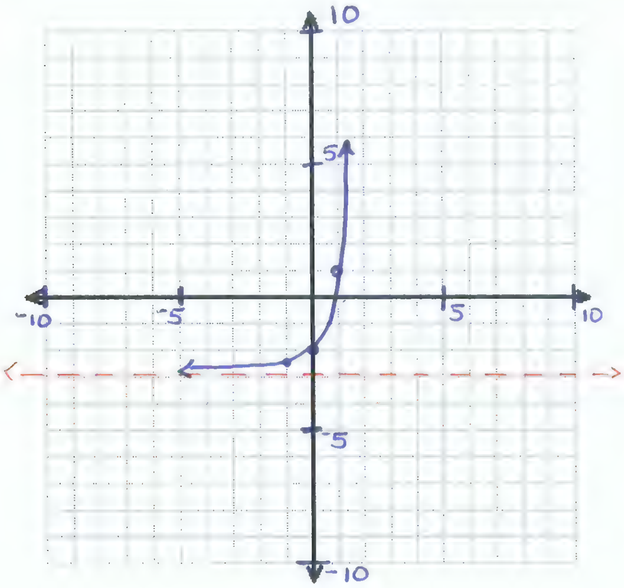


Name Answer Key

PreCalculus; Quiz 3.1 – 3.2

Sketch a graph of the function by finding the asymptote and calculating a few other points. State the domain and range in interval notation. (5 Points Each)

1. $f(x) = 4^x - 3$



Horizontal Asym.

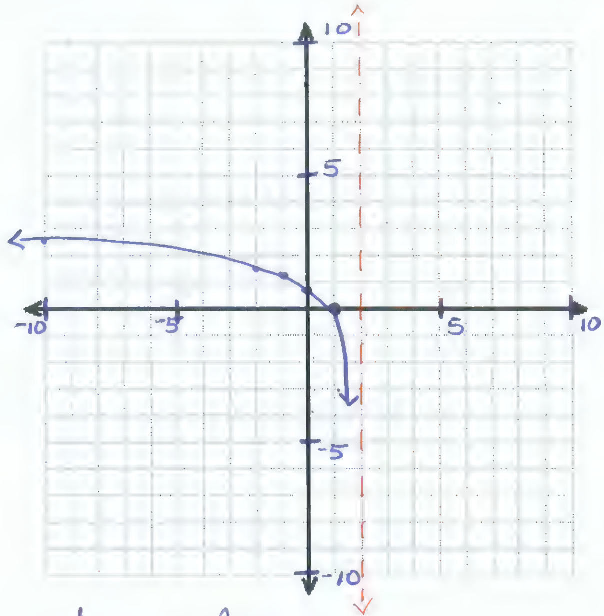
$y = -3$

x	y
-1	-2.75
0	-2
1	1

D: $(-\infty, +\infty)$

R: $(-3, +\infty)$

2. $g(x) = \ln(-x + 2)$



Vertical Asym.

$x = 2$

D: $(-\infty, 2)$

R: $(-\infty, +\infty)$

x	y
1	0
0	0.7
-1	1.1

Evaluate the exponential function at the given value. (1 Point Each)

3. $f(x) = 3.4^x$ at $x = 1.3$

$f(1.3) = 3.4^{1.3} = \boxed{4.908}$

4. $h(x) = -5.5e^{-x}$ at $x = 200$

$h(200) = -5.5e^{-200}$

$\boxed{= -7.611 \times 10^{-87}}$

Evaluate the following logarithms. Round your answer to three decimal places. (1 Point Each)

5. $\log_4 17$

$\boxed{2.044}$

6. $2 \ln e$

$\boxed{2.000}$

Extra Credit

1. Water boils at $212°F$ $(100°C)$. However, if we change the air pressure, water boils at ***different*** temperatures. The function approximates the temperature T (in Fahrenheit) at which water boils at pressure p (in pounds per square inch): $T = 87.97 + 34.96 \ln p + 7.91\sqrt{p}$
   a. What temperature does water boil at when there is zero pressure? (1 Point)

   $T = 87.97 + 34.96 \ln 0 + 7.91\sqrt{0}$

   ↑
   Error

   No solution

   b. What temperature does water boil at when $p = 30$? (1 Point)

   $T = 87.97 + 34.96 \ln 30 + 7.91\sqrt{30}$

   $T = \boxed{250°F}$

   c. What temperature does water boil at when $p = 74$? (1 Point)

   $T = 87.97 + 34.96 \ln 74 + 7.91\sqrt{74}$

   $T = \boxed{306°F}$

Name Answer Key

Pre-Calculus; Vocabulary Test – Chapter 3

Choose the answer that best completes the sentence. Each word will only be used once. (6 Points)

change of base, extraneous, natural, common, logarithmic function, natural exponential

1. The exponential function $f(x) = e^x$ is called the Natural exponential function, and the base e is called the NATURAL base.
2. The inverse function of the exponential function $f(x) = a^x$ is called the Logarithmic function with base a.
3. The base of the Common logarithmic function is 10.
4. You can evaluate logarithms to any base using the Change of base formula.
5. An extraneous solution does not satisfy the original equation.

Complete the following logarithmic and exponential properties. (6 Points)

6. To solve exponential and logarithmic equations, you can use the following One-to-One and inverse properties.
   a. $a^x = a^y$ if and only if $x = y$.
   b. $\log_a x = \log_a y$ if and only if $x = y$.
   c. $a^{\log_a x} =$ x.
   d. $\log_a a^x =$ x.
7. The inverse properties of logarithms are $\log_a a^x = x$ and $a^{\log_a x} = x$.
8. If $x = e^y$, then $y =$ $\ln x$.

Rewrite the logarithmic function as an exponential function. (1 Point Each)

1. $\log_4 16 = 2$

$16 = 4^2$

2. $\log_9 3 = \frac{1}{2}$

$3 = 9^{1/2}$

Rewrite the exponential function as a logarithmic function. (1 Point Each)

3. $4^3 = 64$

$\log_4 64 = 3$

4. $32^{\frac{1}{5}} = 2$

$\log_{32} 2 = 1/5$

Expand **the logarithmic expression.** (2 Points Each)

5. $\ln ab^5$

$\ln a + 5 \ln b$

6. $\log \frac{x^2 y^3}{z}$

$2 \log x + 3 \log y - \log z$

Condense **the logarithmic expression.** (2 Points Each)

7. $\log_4 y - 3\log_4 x$

$\log_4 \frac{y}{x^3}$

8. $\frac{1}{2}\ln x + 2\ln y - 3\ln(x+3)$

$\ln \frac{y^2\sqrt{x}}{(x+3)^3}$

Solve each equation for x. Round your answer to three decimal places, if necessary. (3 Points Each)

9. $480 = 12e^{4x-1}$

$e^{4x-1} = 40$

$4x - 1 = \ln 40$

$4x = 4.689$

$\boxed{x = 1.172}$

10. $\ln(x+1) - \ln(x-2) = \ln x$

$\frac{x+1}{x-2} = x$

$x + 1 = x^2 - 2x$

$x^2 - 3x - 1 = 0$

$x = \frac{3 \pm \sqrt{9+4}}{2} = \boxed{3.303}, \cancel{-0.303}$

11. $3 - \log_4(x-2) = 6$

$-\log_4(x-2) = 3$

$\log_4(x-2) = -3$

$x - 2 = 4^{-3}$

$\boxed{x = 2.016}$

12. $2 - 8e^{-x} = 4$

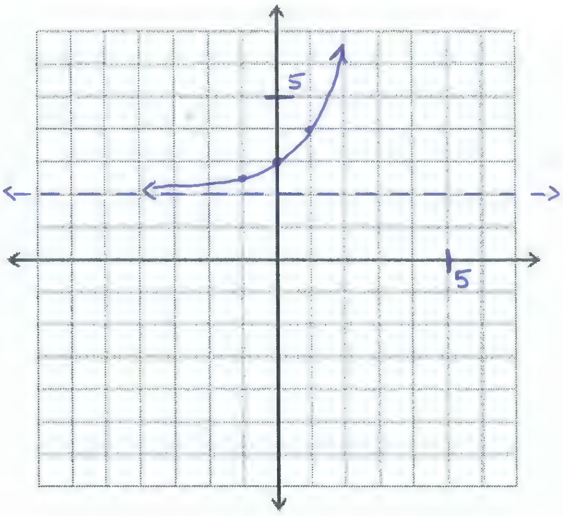
$-8e^{-x} = 2$

$e^{-x} = -\frac{1}{4}$

$\boxed{\text{NO SOLUTION}}$

Graph the following functions by finding the asymptote and plotting a few extra points. State the domain and range in interval notation. (5 Points Each)

13. $f(x) = 2^x + 2$



HORIZONTAL Asym.

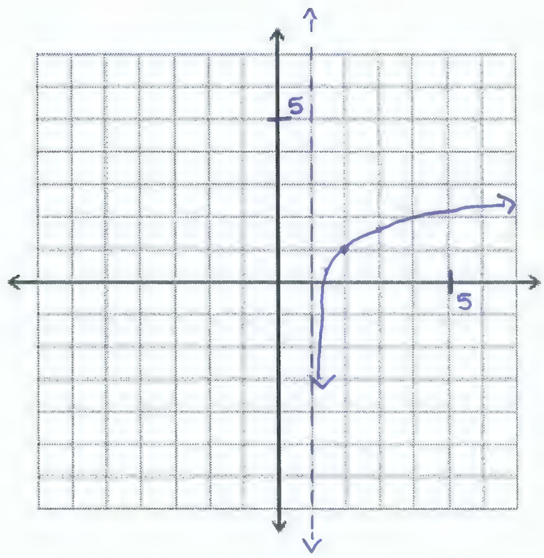
$y = 2$

x	y
-1	2.5
0	3
1	4

D: $(-\infty, +\infty)$

R: $(2, +\infty)$

14. $f(x) = \ln(x - 1) + 1$



VERTICAL Asym.

$x = 1$

x	y
1.5	0.3
2	1
3	1.7

D: $(1, +\infty)$

R: $(-\infty, +\infty)$

15. Johnson wants to open a bank account. He has chosen the option of 7.2% interest compounded quarterly ($n = 4$). Johnson wants to save 120,000 NT and would like to do it in 15 years. How much money should he put in the account to start? Round your answer appropriately. (3 points)

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$120{,}000 = P\left(1 + \frac{.072}{4}\right)^{4(15)}$

$120{,}000 = 2.917P$

$P = 41144.7$

START WITH 41,145 NTD

16. Vickie's bank called and told her that they were changing the interest rate on her account. She currently has 5,600 NT and figures out that in six years she will have 10,300 NT. If her interest is compounded continuously, what is her new interest rate? (3 points)

$$A = Pe^{rt}$$

$10300 = 5600e^{6r}$

$1.84 = e^{6r}$

$\ln 1.84 = 6r$

$r = 0.102$

$\boxed{\text{New interest rate is } 10.2\%}$

Extra Credit

1. Simplify i^{99}. (1 Point)

$i^{99} = i^{3} = -i$

2. On her 23rd birthday, Jessica decided to live by the phrase "an apple a day keeps the doctor away." On her 67th birthday, she calculates the number of apples she has eaten since her 23rd birthday. Since a bushel contains approximately 126 apples, how many bushels of apples has Jessica eaten? (3 Points)

$67 - 23 = 44 \times 365 = 16060$ apples $\div 126 = 127.46$

$\boxed{\text{128 bushels of apples}}$

Name Answer Key

PreCalculus; Test – Chapter 09A

Identify the conic section and sketch the graph. Identify the following for each conic section (20 Points):

Circle – center, radius

Ellipse – center, vertices, foci

Parabola – vertex, focus, directrix

Hyperbolas – center, vertices, foci, asymptotes

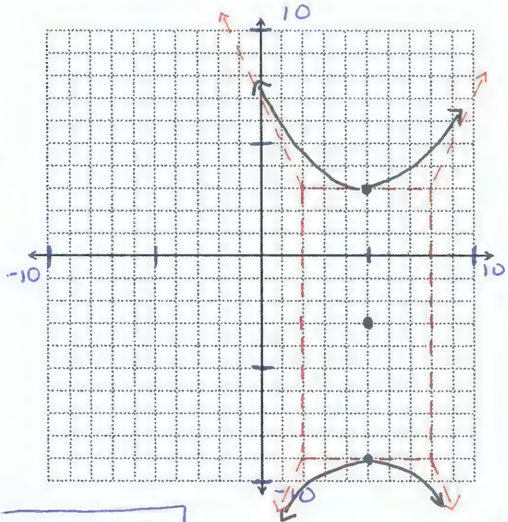
1. $\frac{(y+3)^2}{36} - \frac{(x-5)^2}{9} = 1$

$c^2 = a^2 + b^2$

$c^2 = 36 + 9$

$c^2 = 45$

$c = \pm 3\sqrt{5}$



$\boxed{\text{Hyperbola}}$

Center: $(5, -3)$

Vertices: $(5, -9)$, $(5, 3)$

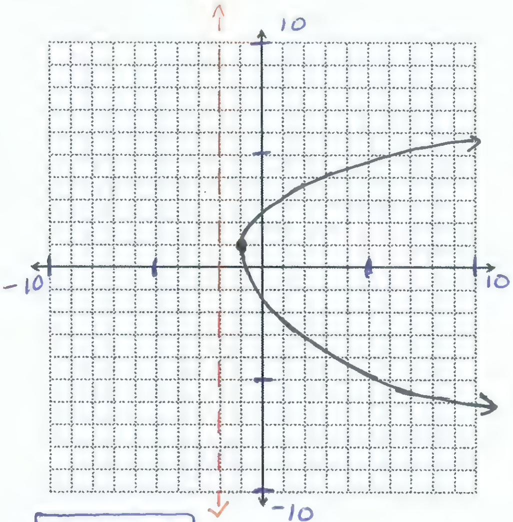
Foci: $(5, -3 \pm \sqrt{45})$ $\pm 3\sqrt{5}$

Asymptotes: $y = -3 \pm 2(x - 5)$

2. $(y-1)^2 = 4(x+1)$

$4p = 4$

$p = 1$



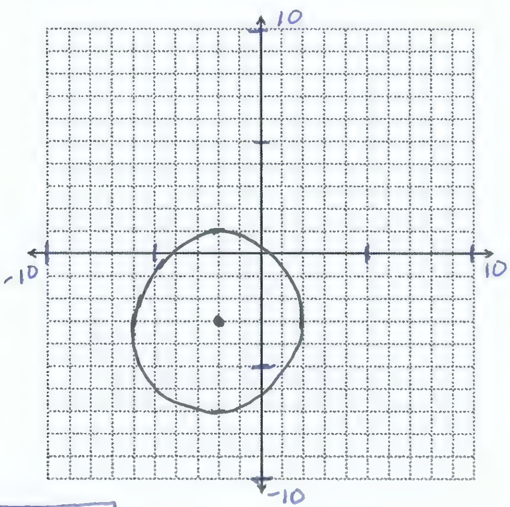
$\boxed{\text{PARABOLA}}$

Vertex: $(-1, 1)$

Focus: $(0, 1)$

Directrix: $x = -2$

3. $(x+2)^2 + (y+3)^2 = 16$



$\boxed{\text{CIRCLE}}$

Center: $(-2, -3)$

Radius: 4

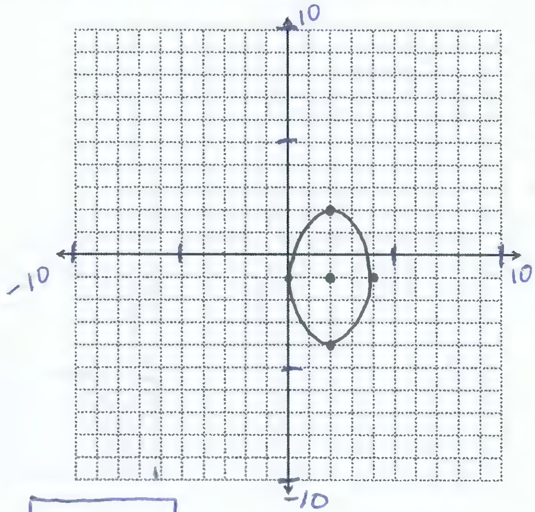
4. $\frac{(x-2)^2}{4} + \frac{(y+1)^2}{9} = 1$

$c^2 = a^2 - b^2$

$c^2 = 9 - 4$

$c^2 = 5$

$c = \pm\sqrt{5}$



$\boxed{\text{Ellipse}}$

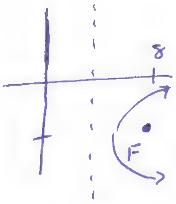
Center: $(2, -1)$

Foci: $(2, -1 \pm \sqrt{5})$

Vertices: $(2, -4)$, $(2, 2)$

Write the given conic section in standard form.

5. Find the standard form of the equation of the parabola with focus at $(8, -2)$ and directrix at $x = 4$.



$(y-k)^2 = 4p(x-h)$

VERTEX
$(6, -2)$

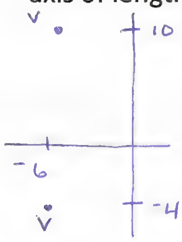
$x = \frac{8+4}{2} = 6$

$\boxed{(y+2)^2 = 8(x-6)}$

$p = 2$

$4p \to 8$

6. Find the standard form of the equation of the ellipse with vertices at $(-6, 10)$ and $(-6, -4)$ and a minor axis of length 8.



$\frac{(x-h)^2}{B^2} + \frac{(y-k)^2}{A^2} = 1$

$\boxed{\frac{(x+6)^2}{16} + \frac{(y-3)^2}{49} = 1}$

$2B = 8$ $\quad a = 7$

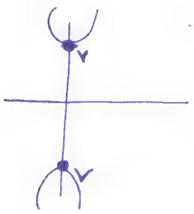
$B = 4$

Center

$\left(\frac{-6+-6}{2}, \frac{10+(-4)}{2}\right)$

$(-6, 3)$

7. Find the standard form of the equation of the hyperbola with vertices at $(0, \pm 3)$ and asymptotes $y = \pm\frac{3}{2}x$.



$\frac{(y-k)^2}{A^2} - \frac{(x-h)^2}{B^2} = 1$

$\boxed{\frac{y^2}{9} - \frac{x^2}{4} = 1}$

Center
$(0, 0)$

$a = 3$

$B = 2$

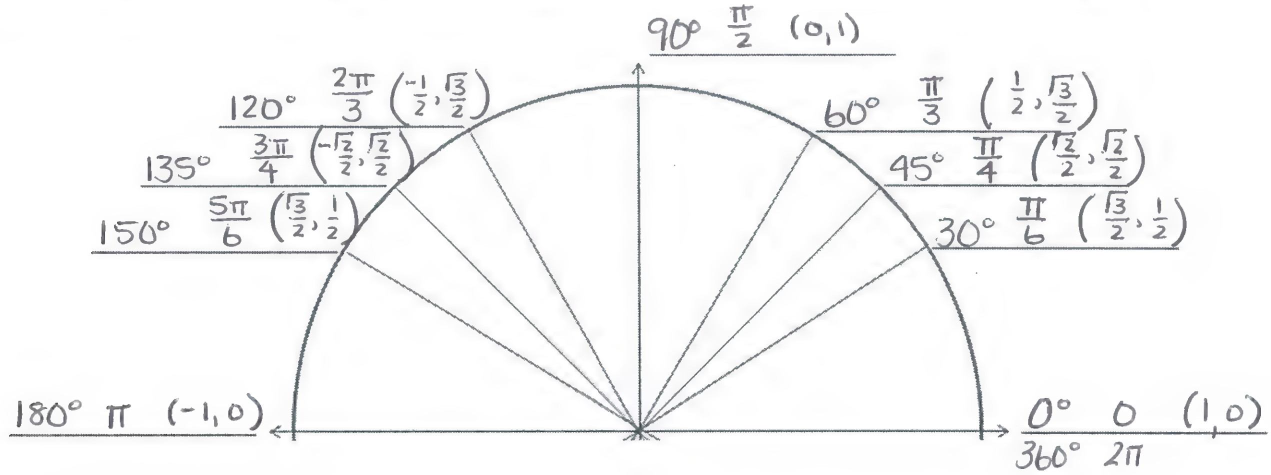
Name Answer Key

PreCalculus; Quiz Unit Circle

The use of a calculator or your unit circle is strictly prohibited!!!

Complete the unit circle below. List the following: degrees, radians, and xy-coordinates. (20 points)

1.



Evaluate (if possible) the six trigonometric functions of the real number t. (6 points)

2. $t = \frac{5\pi}{4}$

$\sin t = \frac{-\sqrt{2}}{2}$ $\quad$ $\csc t = -\sqrt{2}$

$\cos t = \frac{-\sqrt{2}}{2}$ $\quad$ $\sec t = -\sqrt{2}$

$\tan t = 1$ $\quad$ $\cot t = 1$

3. $t = -\frac{7\pi}{6}$

$\sin t = \frac{1}{2}$ $\quad$ $\csc t = 2$

$\cos t = \frac{\sqrt{3}}{2}$ $\quad$ $\sec t = \frac{2\sqrt{3}}{3}$

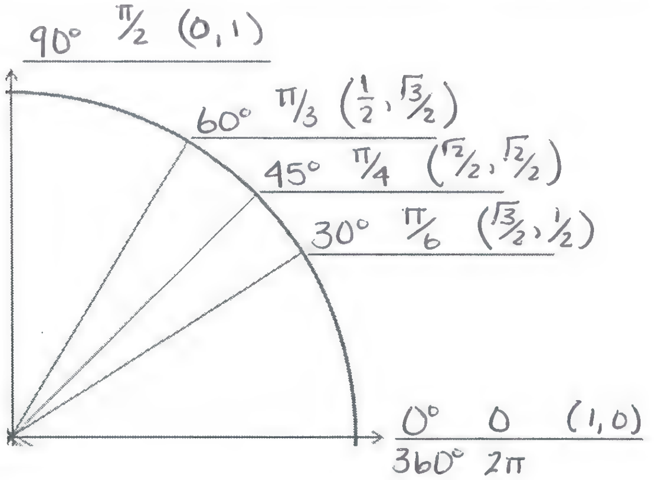
$\tan t = \frac{\sqrt{3}}{3}$ $\quad$ $\cot t = \sqrt{3}$

Name ANSWER KEY

PreCalculus; Quiz 4.1 – 4.4

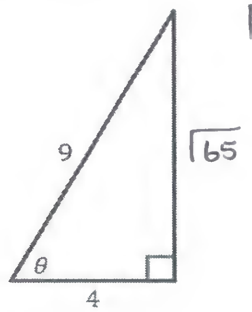
Complete the unit circle below. List the following: degrees, radians, and xy-coordinates.

1.



Find the exact values of the sine, cosine, and tangent functions of the angle θ.

2.



$\sqrt{9^2-4^2}$

$\sin\theta = \frac{\sqrt{65}}{9}$

$\cos\theta = \frac{4}{9}$

$\tan\theta = \frac{\sqrt{65}}{4}$

Use trigonometric identities to transform one side of the equation into the other.

3. $\csc\theta \tan\theta = \sec\theta$

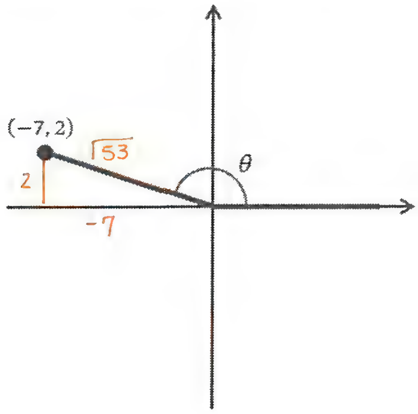
$\frac{1}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta} =$

$\frac{1}{\cos\theta} =$

$\sec\theta = \sec\theta$ ✓

The given point is on the terminal side of angle θ in standard position. Determine the exact values of the six trigonometric functions of the angle.

4.



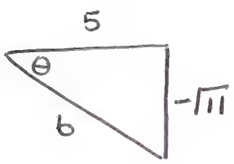
$\sin\theta = \frac{2\sqrt{53}}{53}$ $\csc\theta = \frac{\sqrt{53}}{2}$

$\cos\theta = \frac{-7\sqrt{53}}{53}$ $\sec\theta = \frac{\sqrt{53}}{-7}$

$\tan\theta = \frac{2}{-7}$ $\cot\theta = \frac{-7}{2}$

Find the values of the other five trigonometric functions of θ satisfying the given conditions.

5. $\sec\theta = \frac{6}{5}$, $\tan\theta < 0$



$\sin\theta = \frac{-\sqrt{11}}{6}$ $\csc\theta = \frac{-6\sqrt{11}}{11}$

$\cos\theta = \frac{5}{6}$ $\sec\theta = \frac{6}{5}$

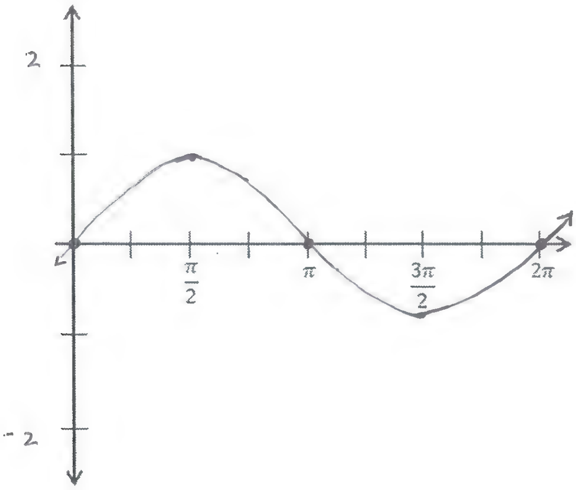
$\tan\theta = \frac{-\sqrt{11}}{5}$ $\cot\theta = \frac{-5\sqrt{11}}{11}$

Name Answer Key

PreCalculus; Quiz 4.5 – 4.6

Graph each of the trigonometric functions on the interval $[0, 2\pi]$.

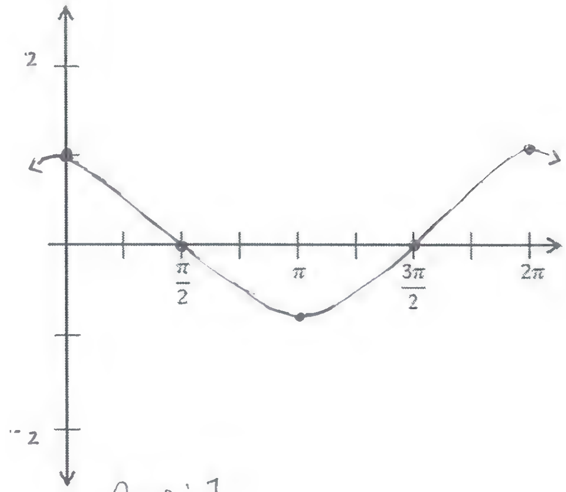
1. $y = \sin x$



Amp. = 1

Per = 2π

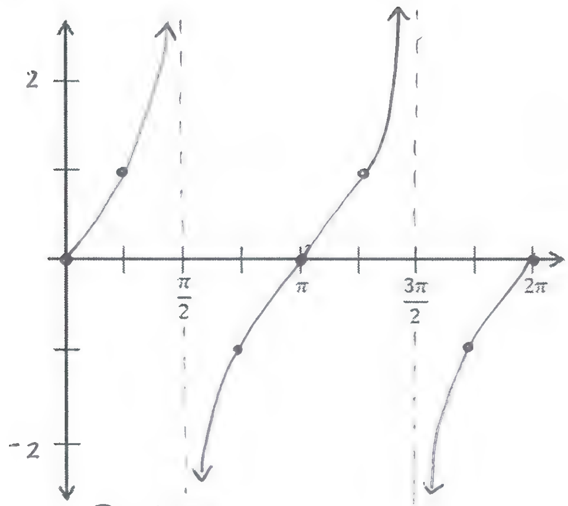
2. $y = \cos x$



Amp: 1

per = 2π

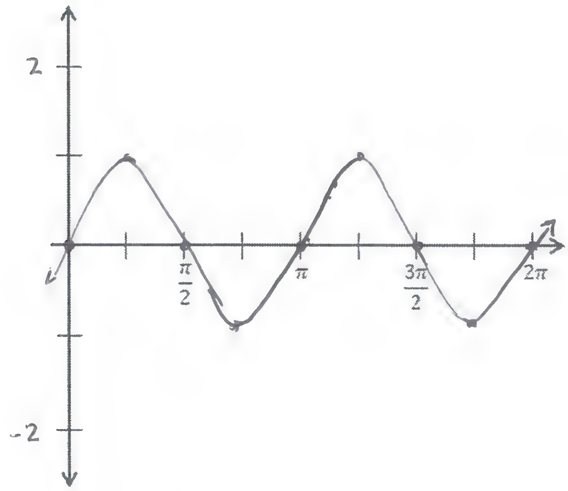
3. $y = \tan x$



Per: π

Asymptote: $x = \frac{\pi}{2}, \frac{3\pi}{2}$

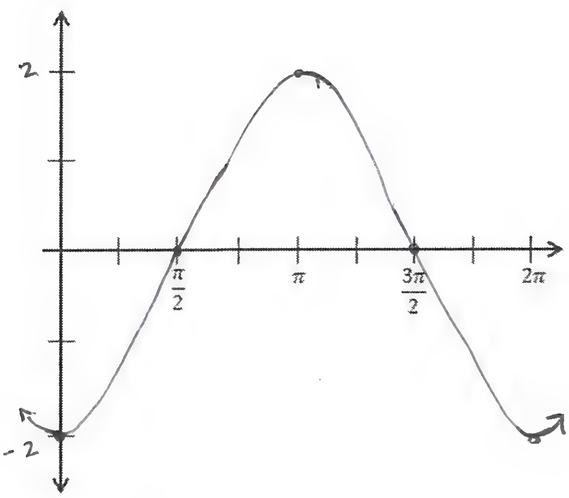
4. $y = \sin 2x$



Amp: 1

per = π

5. $y = -2\cos x$

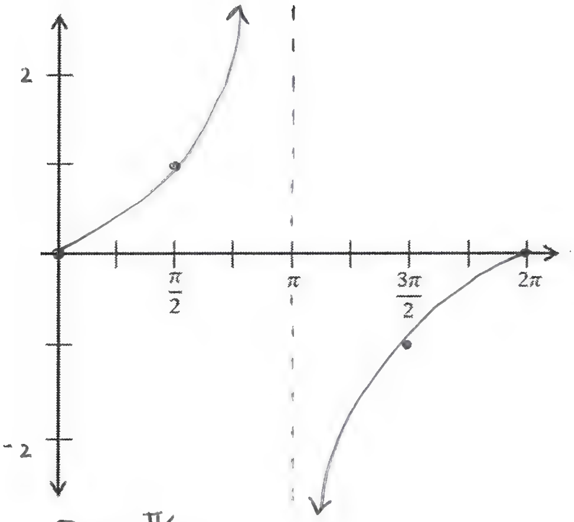


Per $= 2\pi$

Amp: 2

reflect x-axis

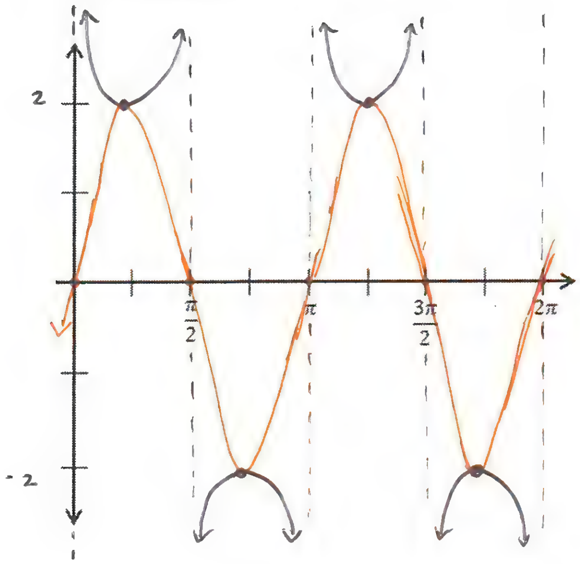
6. $y = \tan\frac{x}{2}$



Per $= \pi/2$

Asymptotes: $x = \pi$

7. $y = 2\csc 2x$

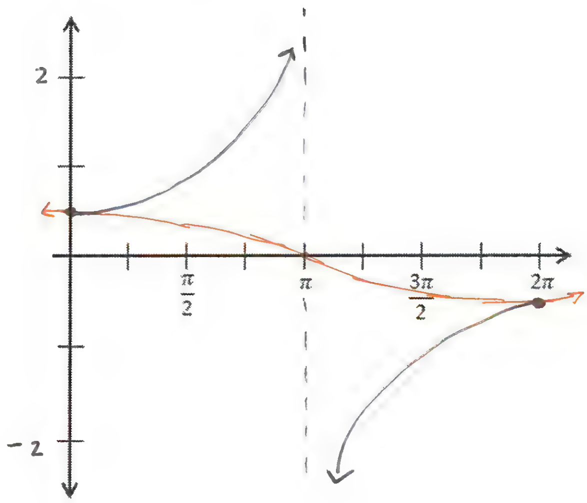


$y = 2\sin 2x$

Amp: 2

Per $= \pi$

8. $y = \frac{1}{2}\sec\frac{x}{2}$



$y = \frac{1}{2}\cos\frac{x}{2}$

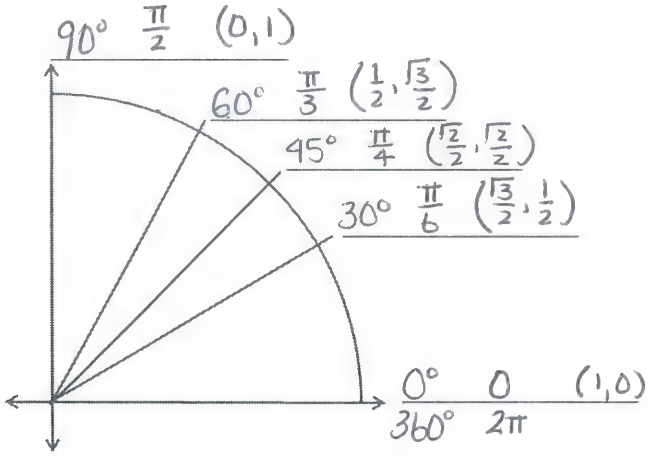
per $= 4\pi$

Amp: $\frac{1}{2}$

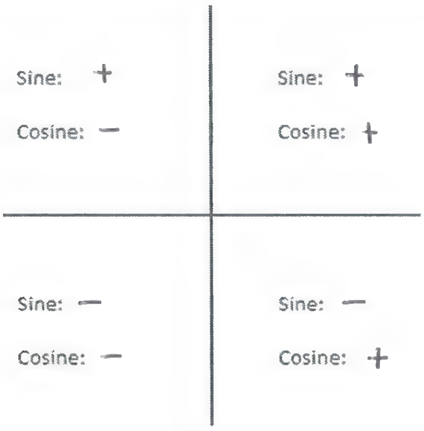
Name Answer Key

Pre-Calculus; Chapter 4 Test (NO Calculator)

1. Complete the first quadrant of the unit circle below. Give each angle in degrees, radians, and the point (x, y). (5 Points)



2. Given the coordinate plane below, indicate the sign (positive or negative) of sine and cosine. (4 Points)



Evaluate the following equations. Check your signs!! (6 Points)

3. $y = \sin \pi$

$\boxed{y = 0}$

4. $y = \cos\left(\frac{\pi}{2}\right)$

$\boxed{y = 0}$

5. $y = \tan\left(\frac{\pi}{6}\right)$

$\boxed{y = \frac{\sqrt{3}}{3}}$

6. $y = \sec\left(\frac{\pi}{3}\right)$

$\boxed{y = 2}$

7. $y = \csc\left(\frac{7\pi}{6}\right)$

$\boxed{y = -2}$

8. $y = \cot\left(-\frac{\pi}{2}\right)$

$\boxed{y = 0}$

Evaluate the following equations on the correct interval. (2 Points)

9. $\arcsin\left(\frac{1}{2}\right) =$ 30°
$\pi/6$

10. $\cos^{-1}(1) =$
0°, 0 rad.

Name ANSWER KEY

Pre-Calculus; Vocabulary Test – Chapter 4

Multiple Choice: Choose the best answer and write it in the blank provided. (9 Points)

C 1. What value below is a vertical asymptote of the function $f(\theta) = \tan\theta$?

a. 0 b. $\frac{\pi}{4}$ c. $\frac{\pi}{2}$ d. π

C 2. The reciprocal function of $f(\theta) = \sin\theta$ is ____________.

a. $f(\theta) = \cos\theta$ b. $f(\theta) = \sec\theta$ c. $f(\theta) = \csc\theta$ d. $f(\theta) = \cot\theta$

D 3. There are ______ quadrants in the unit circle.

a. 1 b. 2 c. 3 d. 4

A 4. The function $f(\theta) = \tan\theta$ can also be written as which of the following?

a. $f(\theta) = \frac{\sin\theta}{\cos\theta}$ b. $f(\theta) = \frac{\cos\theta}{\sin\theta}$ c. $f(\theta) = \frac{1}{\tan\theta}$ d. $f(\theta) = \frac{1}{\cos\theta}$

D 5. What is the point on the unit circle at the angle $\frac{4\pi}{3}$?

a. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ b. $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ c. $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ d. $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

D 6. The period of $f(\theta) = \sin\theta$.

a. 0 b. $\frac{\pi}{2}$ c. π d. 2π

B 7. The angle $\frac{5\pi}{6}$ is in quadrant ______.

a. I b. II c. III d. IV

B 8. The ratio of the cosine function is _______.

a. $\frac{opp}{hyp}$ b. $\frac{adj}{hyp}$ c. $\frac{opp}{adj}$ d. $\frac{adj}{opp}$

B 9. What is the amplitude of $f(x) = -4\sin\left(2x - \frac{\pi}{4}\right)$?

a. -4 b. 4 c. -2 d. 2

Convert the radians to degrees. (1 Point)

1. $\frac{5\pi}{6}$

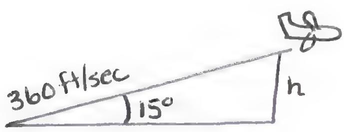
$150°$

Convert the degrees to radians. (1 Point)

2. 355°

$\frac{71\pi}{36}$

3. An airplane takes off at an angle of 15° at a speed of 360 ft/sec. How high is the plane after 2 minutes if it continues in a straight line? (3 Points)

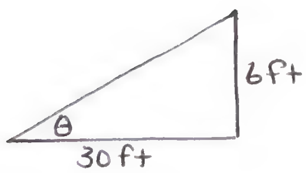


$$\frac{360\text{ ft}}{\text{sec}} \times \frac{60\text{ sec}}{\text{min}} \times 2\text{ min} = 43200\text{ ft}$$

$$\sin 15° = \frac{h}{43200}$$

$$h = \boxed{11{,}181\text{ ft}}$$

4. Angel kicks a ball into the goal during a soccer game. She was 30 feet away from the goal and the ball hit the net at a height of 6 feet. What was the angle of elevation of the ball when it was kicked? (3 Points)

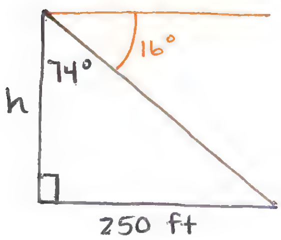


$$\tan\theta = \frac{6}{30}$$

$$\theta = \tan^{-1}\left(\frac{6}{30}\right)$$

$$\boxed{\theta = 11.3°}$$

5. The angle of depression from the top of a lighthouse to a boat out at sea is 16°. The boat is 250 feet away from the light house. How tall is the light house? (3 Points)



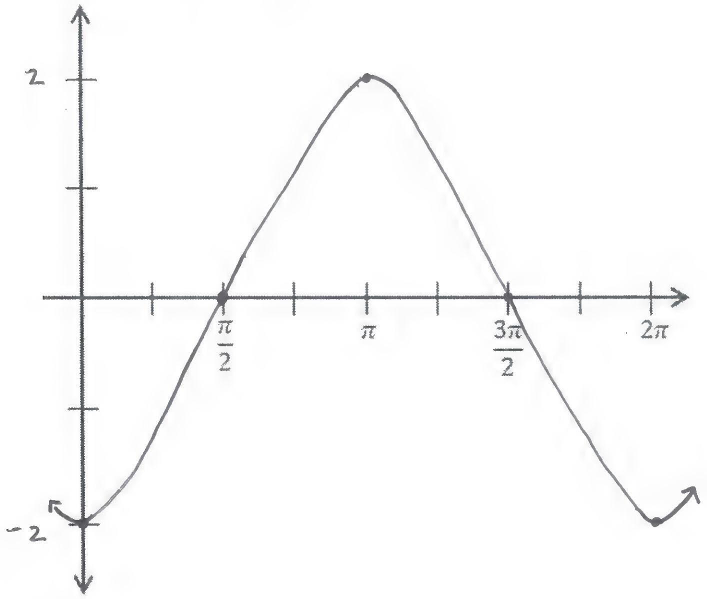
$$\tan 74° = \frac{250}{h}$$

$$h = \frac{250}{\tan 74°}$$

$$\boxed{= 72\text{ ft}}$$

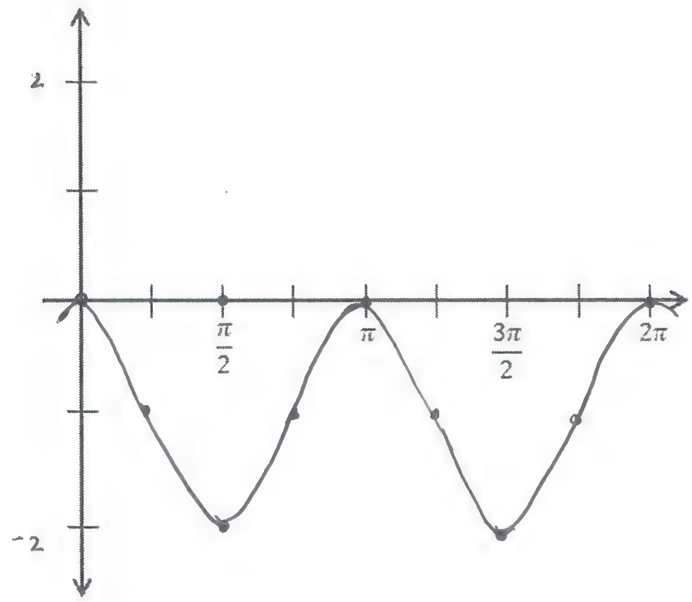
Graph the given functions on the domain $[0, 2\pi]$. Identify the period, amplitude, and any translations.
(5 Points Each)

6. $y = 2\sin\left(x - \frac{\pi}{2}\right)$



Per = 2π
Amp: 2
Shift → $\pi/2$

7. $y = -1 + \cos 2x$



Per = π
Amp: 1
Shift ↓ 1

/10 Name Answer Key

PreCalculus; Quiz 5.1 – 5.2

Use the given values to evaluate (if possible) all six trigonometric functions. (2 Points)

1. $\sec\theta = \sqrt{2}, \sin\theta = -\frac{\sqrt{2}}{2}$

$\cos\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad\quad$ $\csc\theta = -\sqrt{2}$

$\tan\theta = -1$ $\quad\quad$ $\cot\theta = -1$

Use the fundamental identities to simplify the expression. (2 Points Each)

2. $\frac{\tan^2\theta}{\sec^2\theta} = \frac{\sec^2\theta - 1}{\sec^2\theta}$

$= \frac{\sec^2\theta}{\sec^2\theta} - \frac{1}{\sec^2\theta}$

$= 1 - \cos^2\theta$

$\boxed{= \sin^2\theta}$

OR

$\tan^2\theta\cos^2\theta =$

$\boxed{\sin^2\theta}$

3. $\sec^2 x \tan^2 x + \sec^2 x$

$= \sec^2 x(\tan^2 x + 1)$

$= \sec^2 x(\sec^2 x)$

$= \boxed{\sec^4 x}$

OR

$= \sec^2 x(\sec^2 x - 1) + \sec^2 x$

$= \sec^4 x - \sec^2 x + \sec^2 x$

$= \boxed{\sec^4 x}$

Use the fundamental identities to prove each trigonometric identity. (2 Points Each)

4. $\frac{\sec^2\theta}{\sec^2\theta - 1} = \csc^2\theta$

$\frac{\sec^2\theta}{\tan^2\theta} =$

$\sec^2\theta \cdot \cot^2\theta =$

$\frac{1}{\sin^2\theta} = \csc^2\theta$ ✓

5. $\sec^6 x\ (\sec x \tan x) - \sec^4 x\ (\sec x \tan x) = \sec^5 x \tan^3 x$

$(\sec x \tan x)(\sec^6 x - \sec^4 x) =$

$\sec^5 x \tan x(\sec^2 x - 1) =$

$\sec^5 x \tan^3 x =$ ✓

Name Answer Key

Pre-Calculus; Chapter 5 Test

Use the fundamental identities to simplify the expression. (2 Points each)

1. $\sec\alpha \cdot \frac{\sin\alpha}{\tan\alpha}$

$\frac{1}{\cancel{\cos\alpha}} \cdot \frac{\cancel{\sin\alpha}}{1} \cdot \frac{\cancel{\cos\alpha}}{\cancel{\sin\alpha}}$

$\boxed{1}$

2. $\sec^2 x \tan^2 x + \sec^2 x$

$\sec^2 x(\tan^2 x + 1)$

$\sec^2 x(\sec^2 x)$

$\boxed{\sec^4 x}$

3. $\tan x - \frac{\sec^2 x}{\tan x}$

$\frac{\tan^2 x - \sec^2 x}{\tan x}$

$\frac{-1}{\tan x}$

$\boxed{-\cot x}$

4. $\frac{\sin^2 y}{1-\cos y}$

$\frac{1-\cos^2 y}{1-\cos y}$

$\frac{\cancel{(1-\cos y)}(1+\cos y)}{\cancel{1-\cos y}}$

$\boxed{1+\cos y}$

Verify the identity. (3 Points Each)

5. $\sec x - \tan x \sin x = \frac{1}{\sec x}$

$\frac{1}{\cos x} - \frac{\sin^2 x}{\cos x} =$

$\frac{1-\sin^2 x}{\cos x}$

$\frac{\cos^2 x}{\cos x}$

$\cos x = \cos x$ ✓

6. $\tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$

$\tan^2 x(1-\cos^2 x) =$

$\tan^2 x - \tan^2 x \cos^2 x =$

$\tan^2 x - \sin^2 x =$ ✓

7. $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$

$\sin^2\theta + \cancel{2\sin\theta\cos\theta} + \cos^2\theta + \sin^2\theta - \cancel{2\sin\theta\cos\theta} + \cos^2\theta =$

$\sin^2\theta + \cos^2\theta + \sin^2\theta + \cos^2\theta =$

$2(\sin^2\theta + \cos^2\theta) =$

$2 = 2$ ✓

8. $\csc^2\theta \tan^2\theta - 1 = \tan^2\theta$

$\sec^2\theta - 1 =$

$\tan^2\theta =$ ✓

Find all solutions of the equation in the interval $[0, 2\pi)$. (2 Points Each)

9. $\sqrt{2}\sin x + 1 = 0$

$\sin x = \frac{-1}{\sqrt{2}}$ $\frac{-\sqrt{2}}{2}$

$\boxed{x = \frac{5\pi}{4}, \frac{7\pi}{4}}$

10. $2\sec^2 x + \tan^2 x - 3 = 0$

$2\sec^2 x + (\sec^2 x - 1) - 3 = 0$

$3\sec^2 x = 4$

$\sec^2 x = 4/3$

$\cos^2 x = 3/4$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

11. $\sin^2 x - 2\sin x = 0$

$\sin x(\sin x - 2) = 0$

$\sin x = 0$ $\cancel{\sin x = 2}$

$\boxed{x = 0, \pi}$

12. $\sin 2x = -\frac{\sqrt{3}}{2}$

$2x = \frac{4\pi}{3}$ $2x = \frac{5\pi}{3}$

$\boxed{x = \frac{2\pi}{3}, \frac{5\pi}{6}}$

13. $\sec 4x = 2$

$\cos 4x = \frac{1}{2}$

$4x = \frac{\pi}{3}$ $4x = \frac{5\pi}{3}$

$\boxed{x = \frac{\pi}{12}, \frac{5\pi}{12}}$

14. $2\cos(2x) + \sqrt{2} = 0$

$\cos 2x = \frac{-\sqrt{2}}{2}$

$2x = \frac{3\pi}{4}$ $2x = \frac{5\pi}{4}$

$\boxed{x = \frac{3\pi}{8}, \frac{5\pi}{8}}$

Find the exact value of each expression using the Sum and Difference Formulas. (3 Points Each)

15. $\cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

$\cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$\frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right)$

$\boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$

16. $\sin\frac{5\pi}{12} = \sin 75°$

$\sin 45° \cos 30° + \sin 30° \cos 45°$

$\frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)$

$\boxed{\frac{\sqrt{6} + \sqrt{2}}{4}}$

Use the Sum and Difference, Double Angle, Half-Angle, and Sum-to-Product Formulas to find the solution(s) of the equation in the interval $[0, 2\pi)$. (2 Points Each)

17. $\sin\left(x+\frac{\pi}{3}\right)+\sin\left(x-\frac{\pi}{3}\right)=1$

$2\sin\left(\frac{x+\frac{\pi}{3}+x-\frac{\pi}{3}}{2}\right)\cos\left(\frac{x-\frac{\pi}{3}-x+\frac{\pi}{3}}{2}\right)$

$2\sin x\cos 0=1$

$\sin x=\frac{1}{2}$

$\boxed{x=\frac{\pi}{6},\frac{5\pi}{6}}$

18. $\sin 2x\sin x=\cos x$

$(2\sin x\cos x)\sin x-\cos x=0$

$\cos x(2\sin^2 x-1)=0$

$\cos x=0$

$\boxed{\cancel{x=\frac{\pi}{2},\frac{3\pi}{2}}}$

Extraneous solutions.

$\sin^2 x=\frac{1}{2}$

$\sin x=\pm\frac{\sqrt{2}}{2}$

$\boxed{x=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}}$

Write the Product as a Sum or Difference, OR write the Sum or Difference as a Product. (1 Point Each)

19. $6\sin\frac{\pi}{3}\cos\frac{\pi}{3}$

any of these count.

$6\left[\frac{1}{2}\left[\sin\frac{2\pi}{3}+\sin 0\right]\right]$

$3\left[\sin\frac{2\pi}{3}+0\right]$

$\frac{3\sqrt{3}}{2}$

20. $\cos\left(\theta+\frac{\pi}{2}\right)-\cos\left(\theta-\frac{\pi}{2}\right)$

$-2\sin\left(\frac{\theta+\frac{\pi}{2}+\theta-\frac{\pi}{2}}{2}\right)\sin\left(\frac{\theta+\frac{\pi}{2}-\theta+\frac{\pi}{2}}{2}\right)$

$-2\sin\theta\sin\frac{\pi}{2}$

$-2\sin\theta$

Extra Credit

1. (1 Point) Rewrite the expression as a single logarithm and simplify the result:

$$\ln|\csc\theta|+\ln|\tan\theta|$$

$\ln|\csc\theta\tan\theta|$

$\ln|\sec\theta|$

2. (2 Points) Write a sum formula for the following expression:

$$\cos(u+v+w)$$

$\cos u\cos v\cos w-\sin u\sin v\cos w-\sin u\sin v\sin w-\cos u\sin v\sin w.$

Name Answer Key

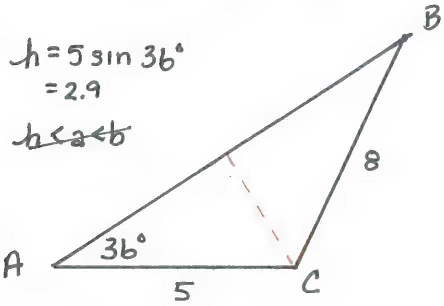
PreCalculus; Test – Chapter 06A

Use the Law of Sines or the Law of Cosines to solve the following triangles. Round your answer to one decimal place. (3 Points Each)

1. $A = 36°, \quad a = 8, \quad b = 5$

$h = 5\sin 36°$
$= 2.9$

$\cancel{h < a < b}$



$\dfrac{\sin B}{5} = \dfrac{\sin 36°}{8}$

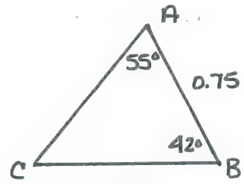
$\boxed{B = 21.6°}$

$\boxed{C = 122.4°}$

$\dfrac{c}{\sin 122.4°} = \dfrac{8}{\sin 36°}$

$\boxed{c = 11.5}$

2. $A = 55°, \quad B = 42°, \quad c = \frac{3}{4}$



$\boxed{C = 83°}$

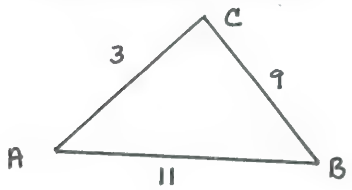
$\dfrac{a}{\sin 55°} = \dfrac{0.75}{\sin 83°}$

$\boxed{a = 0.6}$

$\dfrac{b}{\sin 42°} = \dfrac{0.75}{\sin 83°}$

$\boxed{b = 0.5}$

3. $a = 9, \ b = 3, \ c = 11$



$11^2 = 9^2 + 3^2 - 2(9)(3)\cos C$

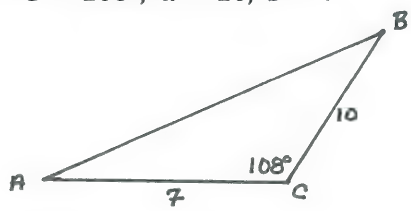
$\boxed{C = 125°}$

$\dfrac{\sin A}{9} = \dfrac{\sin 125°}{11}$

$\boxed{A = 42.1°}$

$\boxed{B = 12.9°}$

4. $C = 108°, \ a = 10, \ b = 7$



$c^2 = 10^2 + 7^2 - 2(10)(7)\cos 108°$

$\boxed{c = 13.9}$

$\dfrac{\sin A}{10} = \dfrac{\sin 108°}{13.9}$

$\boxed{A = 43.2°}$

$\boxed{B = 28.8°}$

Find the area of the triangle with the given information. (1 Point Each)

5. $B = 130°, \quad a = 92, \quad c = 30$

$A = \frac{1}{2}(92)(30)\sin 130°$

$= \boxed{1057.1 \text{ units}^2}$

6. $a = 25, \ b = 35, \ c = 32$

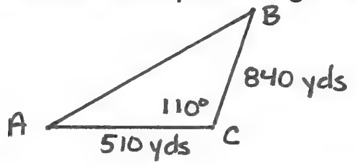
$S = \dfrac{25+35+32}{2} = 46$

$A = \sqrt{46(21)(11)(14)}$

$= \boxed{385.7 \text{ units}^2}$

7. You are buying a piece of land with the following dimensions: $a = 840\, yards$, $b = 510\, yards$, $C = 110°$. The price of land is \$2000 per acre (1 acre = 4840 square yards). For extra credit convert your answer into NT (use \$1US=\$30NT).

a. Draw a sketch representing this problem. (1 Point)



b. How much will this piece of land cost? (2 Points)

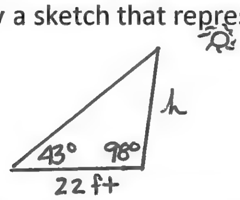
$A = \frac{1}{2}(510)(840)\sin 110° = 201282.1594 \div 4840 = 41.587 \times 2000$

$\boxed{= \$83{,}174.45}$

+1 E.C. ⟶ 2,495,234 NTD

8. A pole tilts *toward* the sun at an 8° angle from the vertical, and it casts a 22-foot shadow. The angle of elevation from the tip of the shadow to the top of the pole is 43°.

a. Draw a sketch that represents this problem. (1 Point)

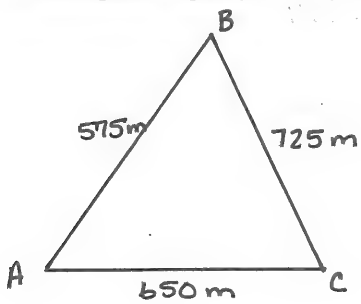


b. How tall is the pole? (2 Points)

$$\frac{h}{\sin 43°} = \frac{22}{\sin 39°}$$

$\boxed{h = 23.8\text{ ft}}$

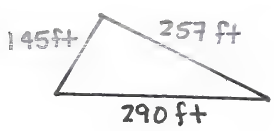
9. A triangular piece of ground has sides of lengths 725 meters, 650 meters, and 575 meters. Find the measure of the ***largest*** angle. (1 Point)



$725^2 = 650^2 + 525^2 - 2(650)(525)\cos A$

$\boxed{B = 72.3°}$

10. The Landau Building in Cambridge, Massachusetts has a triangular-shaped base. The lengths of the sides of the triangular base are 145 feet, 257 feet, and 290 feet. Find the area of the base of the building. For extra credit write your favorite food next to your answer. (2 Points)



$$s = \frac{145+257+290}{2} = 346$$

$$A = \sqrt{346(56)(89)(201)}$$

$$= \boxed{18617.7\text{ ft}^2}$$

Name Answer Key

PreCalculus; Test – Chapter 6B

Find the component form and magnitude of the vector v. (2 Point)

1. Initial Point: $(0, 10)$
   Terminal Point: $(7, 3)$

$\vec{v} = \langle 7-0, 3-10 \rangle$

$\boxed{= \langle 7, -7 \rangle}$

$||\vec{v}|| = \sqrt{7^2 + (-7)^2}$

$= \sqrt{98}$

$\boxed{= 7\sqrt{2}}$

Find (a) $u + v$, (b) $u - v$, (c) $3u$, and (d) $2v + 5u$. (4 Points)

2. $u = 2i - j \quad v = 5i + 3j$

a) $\langle 7, 2 \rangle$

c) $\langle 6, -3 \rangle$

b) $\langle -3, -4 \rangle$

D) $\langle 20, 1 \rangle$

Find a unit vector in the direction of the given vector. (2 Points)

3. $u = \langle 0, -6 \rangle$

$\vec{u}_{unit} = \frac{\vec{u}}{||\vec{u}||} = \frac{\langle 0, -6 \rangle}{\sqrt{36}} = \boxed{\langle 0, -1 \rangle}$

Write the vector as a linear combination of the standard unit vectors i and j. (2 Points)

4. Initial Point: $(-8, 3)$
   Terminal Point: $(1, -5)$

$\langle -9, -8 \rangle$

$\boxed{\vec{v} = -9\vec{i} - 8\vec{j}}$

Find the magnitude and the direction angle of the vector v. (3 Points)

5. $v = 7(\cos 60° \, i + \sin 60° \, j)$

$\left\langle \frac{7}{2}, \frac{7\sqrt{3}}{2} \right\rangle$

$\boxed{||\vec{v}|| = 7}$

$\tan\theta = \frac{7\sqrt{3}/2}{7/2}$

$\boxed{\theta = 60°} = \frac{\pi}{3}$

Find the dot product of u and v. (2 Points)

6. $u = 8i - 7j \quad v = 3i - 4j$

$\vec{u} \cdot \vec{v} = 8(3) + (-7)(-4)$

$\boxed{= 52}$

Use the given vectors to find the indicated quantity. (2 Points Each)

$u = \langle -3, -4 \rangle \quad v = \langle 2, 1 \rangle$

7. $u \cdot u$

$(-3)(-3) + (-4)(-4)$

$\boxed{= 25}$

8. $(u \cdot v)u$

$[\langle -3, -4 \rangle \cdot \langle 2, 1 \rangle] \langle -3, -4 \rangle$

$\boxed{\langle 30, 40 \rangle}$

Find the angle θ between the vectors. (2 Points)

9. $u = \langle 2\sqrt{2}, -4 \rangle \quad v = \langle -\sqrt{2}, 1 \rangle$

$\cos\theta = \dfrac{\langle 2\sqrt{2}, -4 \rangle \langle -\sqrt{2}, 1 \rangle}{\sqrt{24}\ \sqrt{3}}$

$\boxed{\theta = 160.5^\circ} = 2.8$ radians

Determine if the vectors u and v are orthogonal. (2 Points)

10. $u = \langle 8, -4 \rangle \quad v = \langle 5, 10 \rangle$

$\vec{u} \cdot \vec{v} = 8(5) + (-4)(10)$

$= 0$

$\boxed{\text{Yes}}$

Name ANSWER KEY

PreCalculus; Quiz 7.1 – 7.3

Solve the system using either substitution or elimination. (3 Points Each)

1. $\begin{cases} x - y = 6 \\ 3x + 5y = 2 \end{cases}$

$3x - 3y = 18$
$-(3x + 5y = 2)$
$-8y = 16$
$y = -2$
$x = 4$
$\boxed{(4, -2)}$

2. $\begin{cases} y = x - 1 \\ y = (x-1)^3 = x^3 - 3x^2 + 3x - 1 \end{cases}$

$x - 1 = x^3 - 3x^2 + 3x - 1$
$x^3 - 3x^2 + 2x = 0$
$x(x^2 - 3x + 2) = 0$
$x = 0 \qquad x^2 - 3x + 2 = 0$
$y = -1 \qquad (x-1)(x-2) = 0$
$x = 1 \qquad x = 2$
$y = 0 \qquad y = 1$
$\boxed{(0,-1) \quad (1,0) \quad (2,1)}$

3. $\begin{cases} 4x - y^2 = 7 \\ x - y = 3 \rightarrow x = y + 3 \end{cases}$

$4(y+3) - y^2 = 7$
$-y^2 + 4y + 12 = 7$
$-y^2 + 4y + 5 = 0$
$y^2 - 4y - 5 = 0$
$(y-5)(y+1) = 0$
$y = 5 \qquad y = -1$
$x = 8 \qquad x = 2$
$\boxed{(8,5) \quad (2,-1)}$

4. $\begin{cases} x - 4y - z = 3 & Ⓐ \\ 2x - 5y + z = 0 & Ⓑ \\ 3x - 3y + 2z = -1 & Ⓒ \end{cases}$

Ⓐ+Ⓑ
$x - 4y - z = 3$
$+\ 2x - 5y + z = 0$
$3x - 9y = 3 \xrightarrow{\times 5} 15x - 45y = 15$

2Ⓐ+Ⓒ
$2x - 8y - 2z = 6$
$+\ 3x - 3y + 2z = -1$
$5x - 11y = 5 \xrightarrow{\times 3} -(15x - 33y = 15)$

$-12y = 0$
$\boxed{y = 0}$
$3x - 0 = 3$
$\boxed{x = 1}$
$1 - 0 - z = 3$
$\boxed{z = -2}$

$\boxed{(1, 0, -2)}$

Write the partial fraction decomposition for the rational expression. (2 Points Each)

5. $\frac{5x-2}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$

$5x - 2 = A(x-1) + B$
$5x - 2 = Ax - A + B$
$A = 5$
$-A + B = -2$
$B = 3$

$\boxed{\frac{5x-2}{(x-1)^2} = \frac{5}{x-1} + \frac{3}{(x-1)^2}}$

6. $\frac{x^3+x^2+x+2}{x^4+x^2} = \frac{x^3+x^2+x+2}{x^2(x^2+1)}$

$\frac{x^3+x^2+x+2}{x^2(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1}$

$x^3 + x^2 + x + 2 = Ax(x^2+1) + B(x^2+1) + (Cx+D)x^2$
$x^3 + x^2 + x + 2 = Ax^3 + Ax + Bx^2 + B + Cx^3 + Dx^2$

$A + C = 1$
$B + D = 1$
$A = 1$
$B = 2$
$C = 0$
$D = -1$

$\boxed{\frac{x^3+x^2+x+2}{x^4+x^2} = \frac{1}{x} + \frac{2}{x^2} + \frac{-1}{x^2+1}}$

16

7. You want to buy either a wood pellet stove or an electric furnace. The pellet stove costs $2160 and produces heat a cost of $15.15 per 1 million Btu (British thermal units). The electric furnace costs $1250 and produces heat at a cost of $33.25 per 1 million Btu.

   a. Write a function for the total cost y of buying the pellet stove and producing x million Btu of heat. (1 Point)

   $y = 15.15x + 2160$

   b. Write a function for the total cost y of buying the electric furnace and producing x million Btu of heat. (1 Point)

   $y = 33.25x + 1250$

   c. Solve the system of equations. (2 Points)

   $15.15x + 2160 = 33.25x + 1250$

   $910 = 18.1x$

   $\boxed{x = 50.3 \text{ million Btu} \quad y = \$2922.5}$

   d. Interpret the results in the context of the problem. (1 Point)

   At 50.3 million Btu, the two objects will cost the same.

8. In the 2010 Women's NCAA Championship basketball game, the University of Connecticut defeated Stanford University by a score of 53 to 47. Connecticut won by scoring a combination of two-point baskets, three-point baskets, and one-point free throws. The number of two-point baskets was four more than the number of free throws. The number of free throws was three more than the number of three-point baskets. What combination of scoring accounted for Connecticut's 53 points? (3 Points)

$a = 3$ pt
$b = 2$ pt
$c = 1$ pt

$3a + 2b + c = 53$

$b = c + 4 = a + 7$

$c = a + 3$

$3a + 2(a+7) + (a+3) = 53$

$6a + 17 = 53$

$6a = 36$

$\boxed{a = 6 \quad B = 13 \quad c = 9}$

$\boxed{\text{6 3-pt baskets, 13 2-pt baskets, 9 1-pt baskets}}$

/8

Name ANSWER KEY

PreCalculus; Quiz 8.1 – 8.4

Write the first five terms of the sequence. (5 Points each)

1. $a_n = \left(-\frac{2}{3}\right)^{n-1}$ (Begin with $n = 1$.)

$a_1 = 1$

$a_2 = \frac{-2}{3}$

$a_3 = \frac{4}{9}$

$a_4 = \frac{-8}{27}$

$a_5 = \frac{16}{81}$

2. $a_1 = 12$ and $a_{k+1} = a_k + 4$

$a_1 = 12$

$a_2 = 16$

$a_3 = 20$

$a_4 = 24$

$a_5 = 28$

Simplify the factorial expression. (1 Point Each)

3. $\frac{11!\cdot 4!}{4!\cdot 7!}$

$$\frac{11\cdot 10\cdot 9\cdot 8\cdot \cancel{7\cdot 6\ldots}\cdot \cancel{4\cdot 3\cdot 2\cdot 1}}{\cancel{4\cdot 3\cdot 2\cdot 1}\cdot \cancel{7\cdot 6\cdot 5\ldots}}$$

$\boxed{= 7290}$

4. $\frac{n!}{(n+1)!}$

$$\frac{\cancel{n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot 2\cdot 1}}{(n+1)\cdot \cancel{n\cdot(n-1)\cdot\ldots\cdot 2\cdot 1}}$$

$\boxed{\frac{1}{n+1}}$

Write an expression for the *apparent* nth term of the sequence. (Assume n begins with 1.) (1 Point)

5. $2, 5, 10, 17, 26, \ldots$

$n = 1, 2, 3, 4, 5, \ldots$

$a_n = n^2 + 1$

Find a formula for the nth term of the sequence. (1 Point Each)

6. Arithmetic: $a_1 = 5000, d = -100$

$a_n = 5000 - 100(n-1)$

$= 5100 - 100n$

7. Geometric: $a_1 = 4, a_{k+1} = \frac{1}{2}a_k$

$r = \frac{1}{2}$

$a_n = 4\left(\frac{1}{2}\right)^{n-1}$

Write the sequences using sigma notation. (2 Points Each)

8. $\frac{2}{3(1)+1}+\frac{2}{3(2)+1}+\cdots+\frac{2}{3(12)+1}$

$\sum_{n=1}^{12} \frac{2}{3n+1}$

9. $2+\frac{1}{2}+\frac{1}{8}+\frac{1}{32}+\frac{1}{128}+\cdots$

$\sum_{n=1}^{\infty} 2\left(\frac{1}{4}\right)^{n-1}$

Find the sum. (2 Points Each)

10. $\sum_{n=1}^{7}(8n-5)$

$S_7 = \frac{7(3+51)}{2}$

$\boxed{=189}$

11. $\sum_{n=1}^{8} 24\left(\frac{1}{6}\right)^{n-1}$

$S_8 = \frac{24\left(1-(1/6)^8\right)}{1-\frac{1}{6}}$

$\boxed{=28.8}$

Find the indicated value. (1 Point)

12. ${}_9C_3$

84

Use the Binomial Theorem or Pascal's Triangle to expand and simplify. (3 Points each)

13. $(x-2y)^4$ 1 4 6 4 1

$= x^4 + 4x^3(-2y) + 6x^2(-2y)^2 + 4x(-2y)^3 + (-2y)^4$

$\boxed{= x^4 - 8x^3y + 24x^2y^2 - 32xy^3 + 16y^4}$

14. $(3+2i)^3$ 1 3 3 1

$= 3^3 + 3(3)^2(2i) + 3(3)(2i)^2 + (2i)^3$

$= 27 + 54i + 36i^2 + 8i^3$

$= 27 + 54i - 36 - 8i$

$\boxed{= -9 + 46i}$

Useful Formulas

Sec 8.2 – Arithmetic Sequences and Partial Sums

$a_n = a_1 + d(n-1)$ $\quad S_n = \frac{n(a_1+a_n)}{2}$

Sec 8.3 – Geometric Sequences and Series

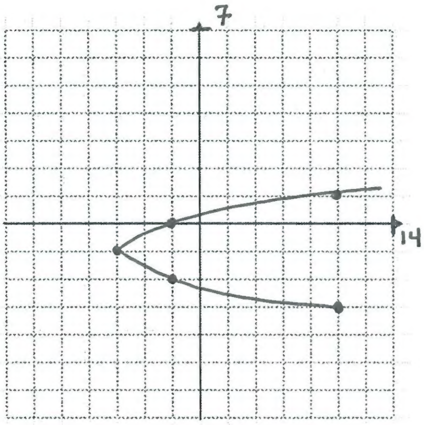
$a_n = a_1 r^{n-1}$ $\quad S_n = \frac{a_1(1-r^n)}{1-r}$ $\quad S = \frac{a_1}{1-r}$

Name Answer Key

Pre-Calculus; Test – Chapter 9B

Sketch the curve represented by the parametric equations. Then eliminate the parameter and write the parametric equations in rectangular form. (4 Points Each)

1. $x = t^2 - 6$, $y = \frac{1}{2}t - 1$, $-4 \le t \le 4$



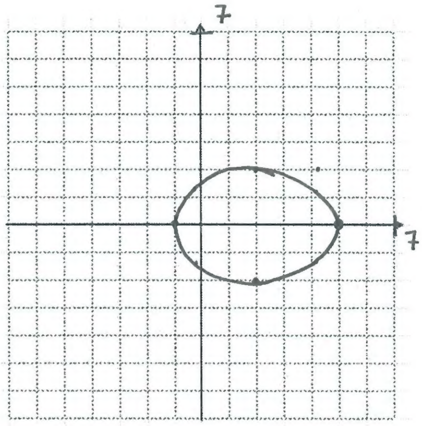
t	-4	-2	0	2	4
x	10	-2	-6	-2	10
y	-3	-2	-1	0	1

$t^2 = x + 6$

$t = \sqrt{x+6}$

$\boxed{y = \frac{1}{2}\sqrt{x+6} - 1}$

2. $x = 2 + 3\cos\theta$, $y = 2\sin\theta$, $0° \le \theta \le 360°$



θ	0	45	90	135	180	225	270	315
x	5	4.1	2	-.1	-1	-.1	2	4.1
y	0	1.4	2	1.4	0	-1.4	-2	-1.4

$\frac{x-2}{3} = \cos\theta$

$\left[\frac{x-2}{3}\right]^2 = \cos^2\theta$

$\sqrt{1 - \left[\frac{x-2}{3}\right]^2} = \sin\theta$

$\boxed{y = 2\sqrt{1 - \left[\frac{x-2}{3}\right]^2}}$

Convert the polar coordinates to rectangular form. (2 Points)

3. $\left(-2, \frac{5\pi}{6}\right)$

$x = -2\cos\left(\frac{5\pi}{6}\right)$

$y = -2\sin\left(\frac{5\pi}{6}\right)$

$\boxed{(\sqrt{3}, -1)}$

Convert the rectangular coordinates to polar form and find one additional representation of this point for $-2\pi \le \theta \le 2\pi$. (3 Points)

4. $(2, -2)$

$\tan\theta = \frac{-2}{2}$

$\theta = -45°$

$r^2 = 2^2 + (-2)^2$

$r = 2\sqrt{2} = \sqrt{8}$

$\boxed{(\sqrt{8}, -45°)}$

$\boxed{(\sqrt{8}, 315°)}$

Convert the rectangular equation to polar form. (3 Points)

5. $x^2 + y^2 - 3x = 0$

$x^2 + y^2 = 3x$

$r^2 = 3r\cos\theta$

$\boxed{r = 3\cos\theta}$

Convert the polar equation to rectangular form. (3 Points)

6. $r = 2\sin\theta$

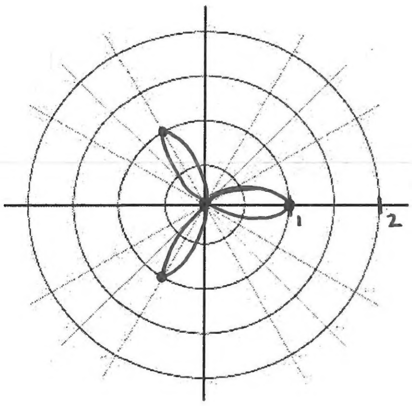
$r^2 = 2r\sin\theta$

$x^2 + y^2 = 2y$

$\boxed{x^2 + y^2 - 2y = 0}$

Sketch the graph of the polar equation. (3 Points)

7. $r = \cos 3\theta$



θ	r
0°	1
30°	0
60°	-1
90°	0
120°	1
150°	0
180°	-1

Useful Information

Polar Equations: $x = r\cos\theta$ $y = r\sin\theta$

$x^2 + y^2 = r^2$ $\tan\theta = \frac{y}{x}$